Non-perturbative determination of collisional broadening and medium induced radiation in QCD plasma

Ismail Soudi

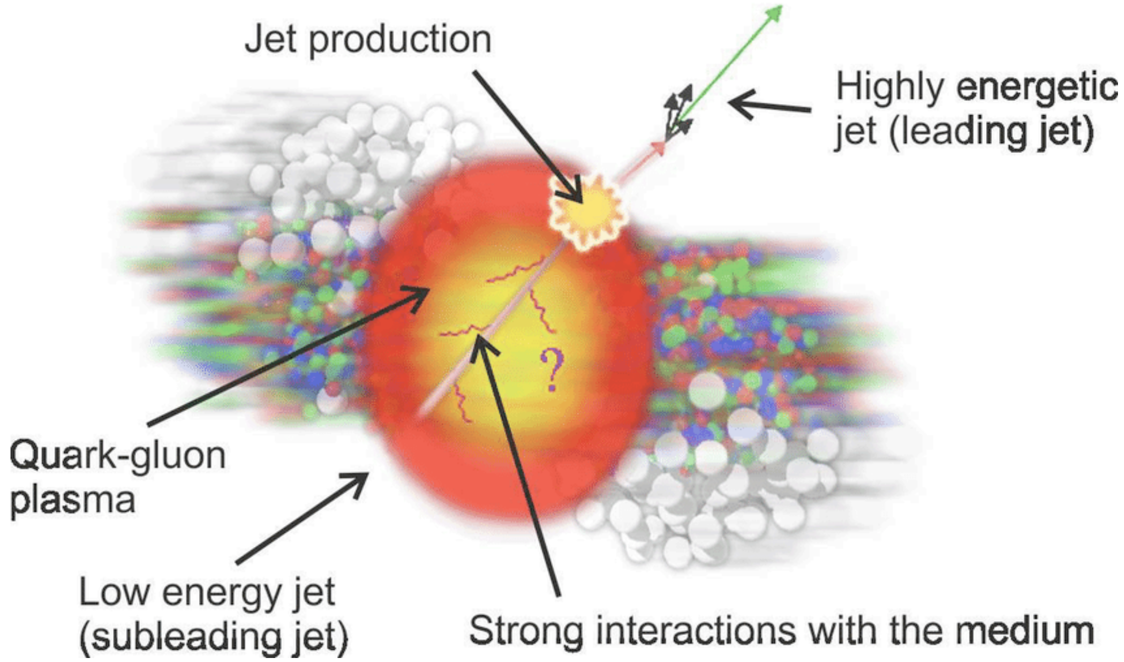
Universität Bielefeld

Based on: G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv: 2105.01679

S. Schlichting, I.S. work in progress

Wayne State University,
May 8 2021

Understanding the QGP

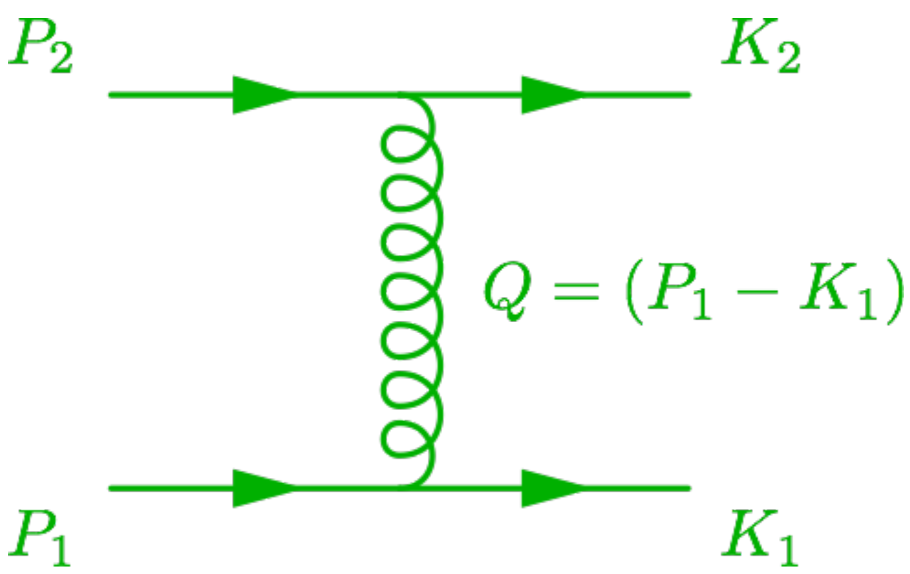


- Quark-Gluon-Plasma (QGP) is formed during Heavy-Ion collisions at RHIC and LHC.
- Different probes of the QGP : collective flow, heavy flavor, EM probes..
- High energetic particles (jets) created at the early stage of the collision can be used as probes to understand the medium.

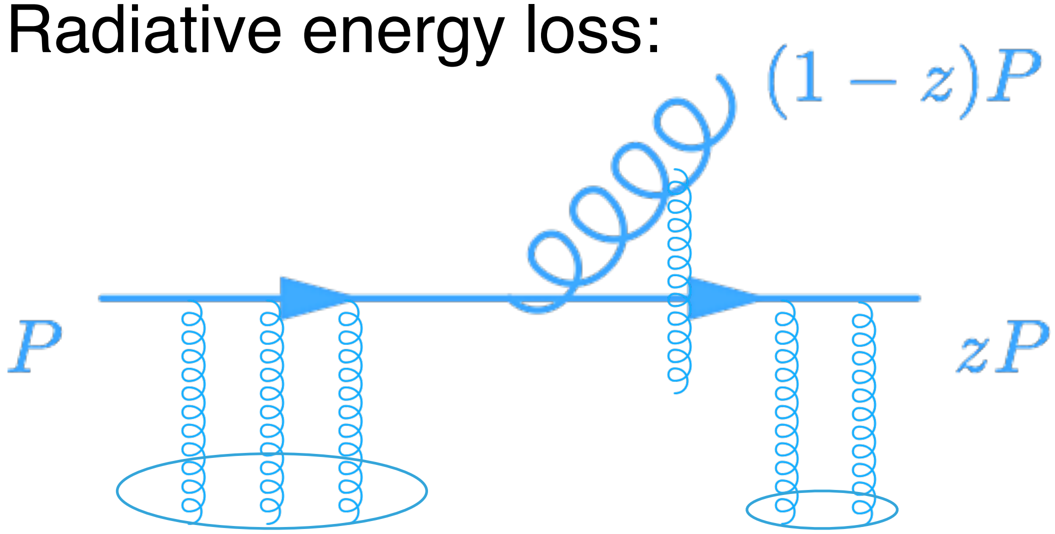
Jet studies

As the high energetic partons traverse the medium they lose energy due to:

- Elastic energy loss:



- Radiative energy loss:



Both require input from the medium
Transverse scattering rate :

$$\lim_{p \to \infty} \frac{d\Gamma(\boldsymbol{p}, \boldsymbol{p} + \boldsymbol{q}_\perp)}{d^2 q_\perp} = \frac{\mathcal{C}(q_\perp)}{(2\pi)^2}.$$

Transverse momentum broadening rate :

In the literature one employs pQCD broadening kernels:

- Static screened color centers -> $C(q) \propto \frac{1}{(q^2 + m_D^2)^2}$
- Dynamics moving charges -> $C(q) \propto \frac{1}{q^2(q^2 + m_D^2)}$
- Multiple soft scattering -> $C(b) \propto \frac{\hat{q}}{4} b^2$

EQCD broadening kernel

Due to the infamous infrared problem of finite temperature QCD

=> perturbative calculation can receive large non-perturbative contribution even at small coupling.

Using effective-field-theory models coupled with lattice gauge calculation one can resolve this problem.

[E. Braaten & A. Nieto, Phys. Rev. Lett. 74, 2164 (1995)]

The collision kernel can be defined in terms of the behavior of certain null Wilson loops

=>For temperatures well above Tc these Wilson loops can be recast in the reduced effective theory of electrostatic QCD (EQCD)

[J. Casalderrey-Solana & D. Teaney, JHEP, vol. 04, p. 039, 2007]

[P. Arnold & W. Xiao Phys.Rev.D 78 (2008), 125008]

EQCD broadening kernel

The kernel was computed in a perturbative expansion in the effective theory of EQCD :

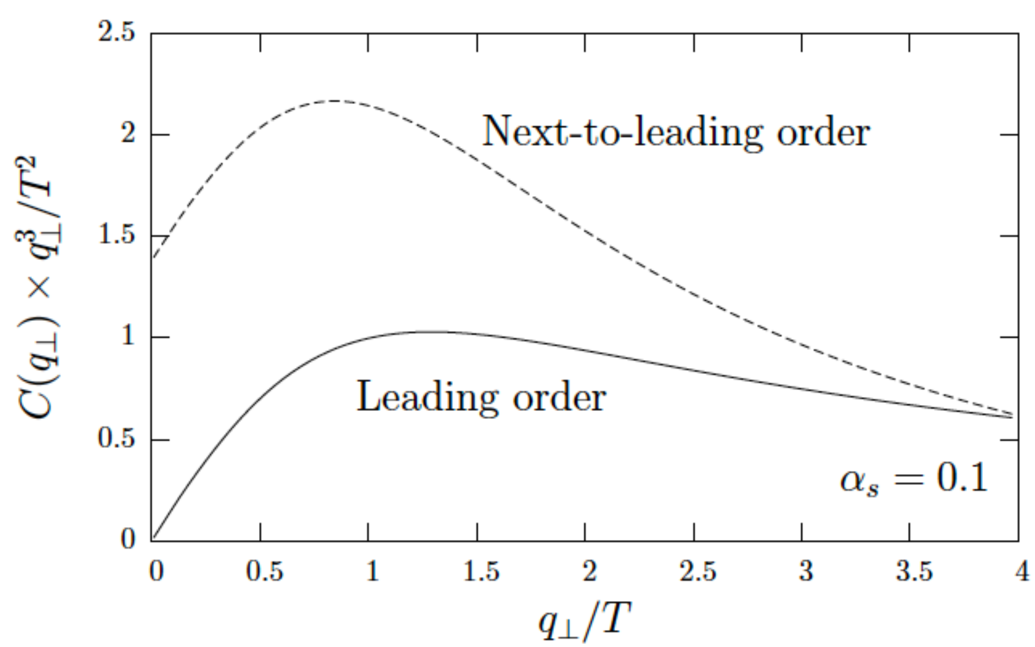
- The LO EQCD kernel

$$C^{\mathrm{LO}}_{\mathrm{QCD}}(q_\perp) = \frac{g_s^4 T^3 C_{\mathrm{R}}}{q_\perp^2 (q_\perp^2 + m_D^2)} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{p - p_z}{p} \left[2 C_{\mathrm{A}} n_{\mathrm{B}}(p) \left(1 + n_{\mathrm{B}}(p')\right) + 4\, N_{\mathrm{f}} T_{\mathrm{f}}\, n_{\mathrm{F}}(p) \left(1 - n_{\mathrm{F}}(p')\right)\right] ,$$

$$= g_s^2 T C_{\mathrm{R}} \begin{cases} \frac{m_{\mathrm{D}}^2 - g_s^2 T^2 C_{\mathrm{A}} \frac{q_\perp}{16T}}{q_\perp^2 (q_\perp^2 + m_{\mathrm{D}}^2)} , & q_\perp \ll g_s T , \\ \frac{g_s^2 T}{q_\perp^4} \mathcal{N} , & q_\perp \gg g_s T , \end{cases}$$

[P. Arnold & W. Xiao Phys.Rev.D 78 (2008), 125008]

- NLO corrections :

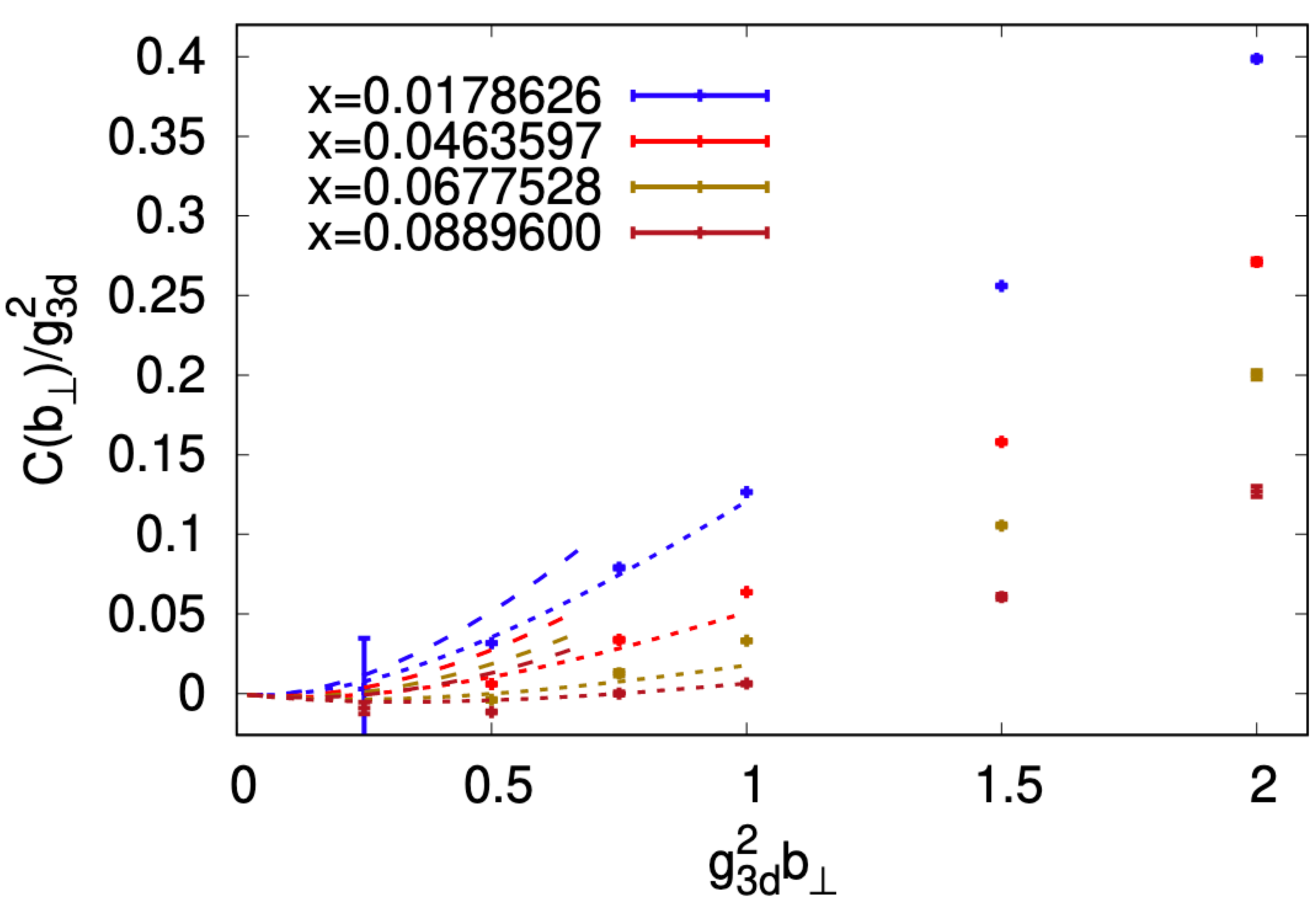


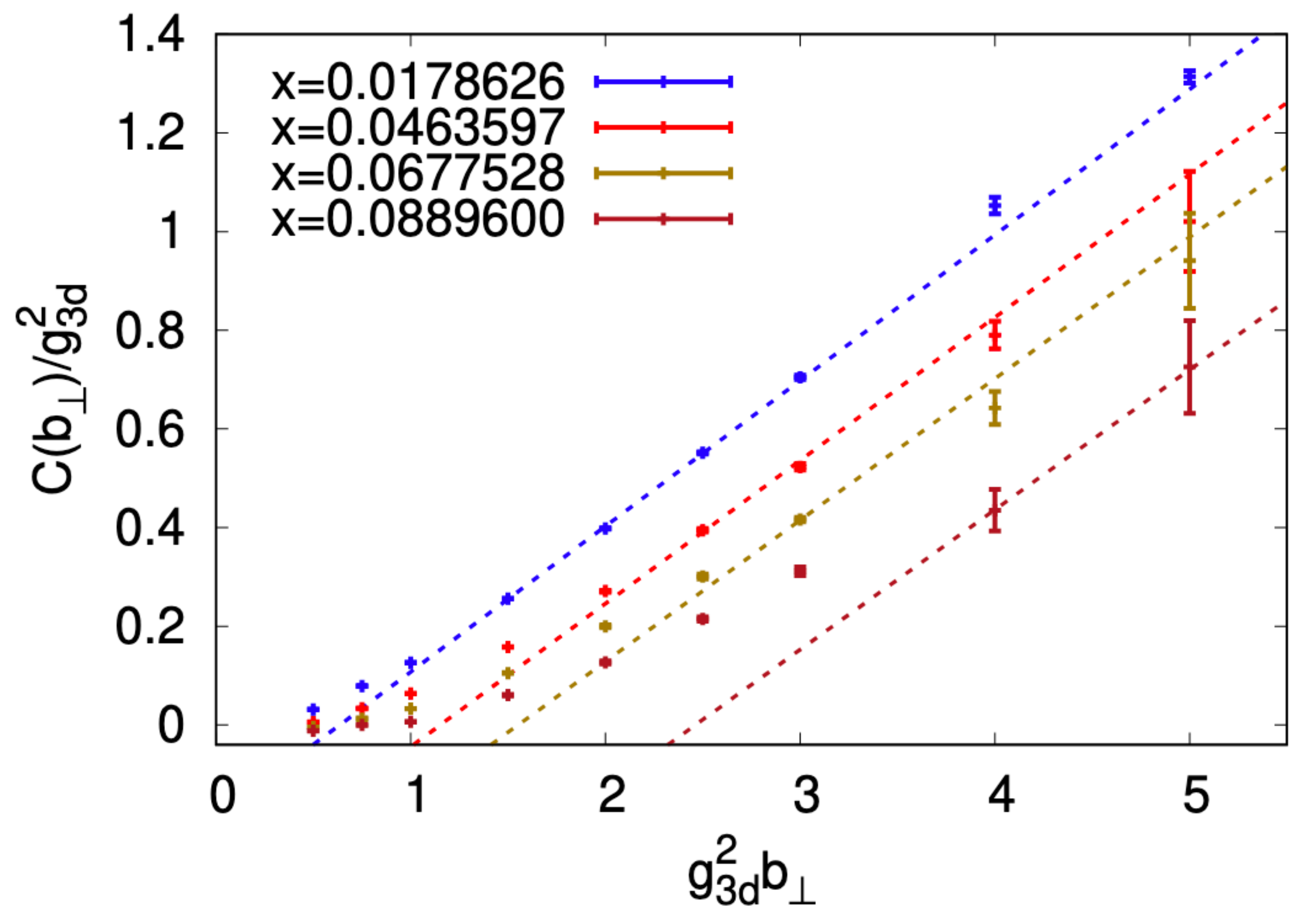
[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

EQCD broadening kernel

Beyond the perturbative result, lattice extracted non-perturbative contribution were computed

$$C^{\mathrm{latt}}_{\mathrm{EQCD}}(b_\perp) \equiv \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} \left(1 - e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}\right) C(q_\perp) \, .$$





This result here is for the broadening kernel in EQCD which need to be matched to QCD

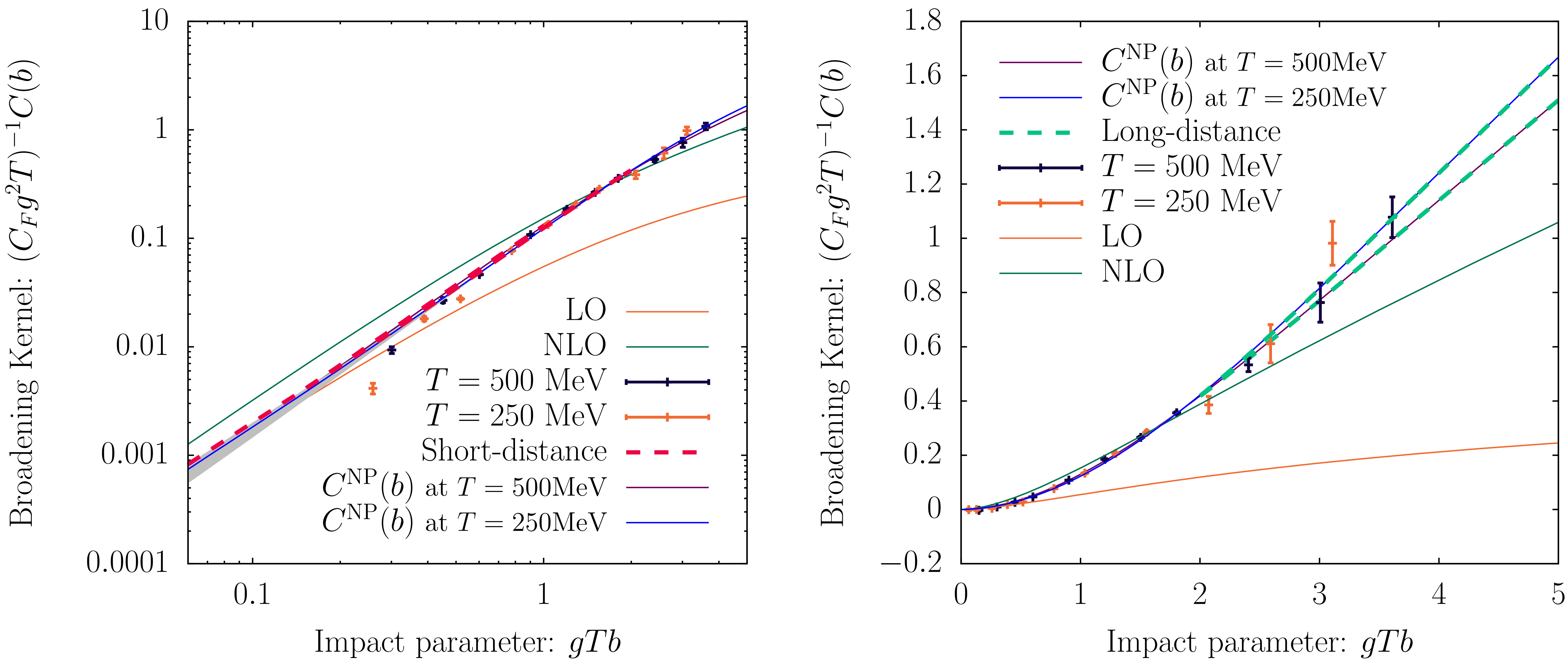
[G.D. Moore & N. Schlusser Phys.Rev.D 101 (2020) 1, 014505]

Non-perturbative broadening kernel

Since EQCD is a low-energy effective theory for QCD they should both agree in the IR regime but in the UV they can be different.

In order to ensure the right UV behavior while keeping the IR behavior from the lattice result we write the full kernel:

$$C_{\rm QCD}(b_\perp) \approx \left(C^{\rm pert}_{\rm QCD}(b_\perp) - C^{\rm pert}_{\rm EQCD}(b_\perp)\right) + C^{\rm latt}_{\rm EQCD}(b_\perp)\,.$$



[G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv:2105.01679]

Non-perturbative broadening kernel

Long-distance behavior :

The kernel follows an area-law with sub-leading logarithm corrections

$$\frac{C_{\mathrm{QCD}}}{g_{3\mathrm{d}}^2}(b_\perp) \xrightarrow{b_\perp \gg 1/g_{3\mathrm{d}}^2} \boxed{A + \frac{\sigma_{\mathrm{EQCD}}}{g_{3\mathrm{d}}^4} g_{3\mathrm{d}}^2 b_\perp} + \frac{g_{\mathrm{s}}^4 C_{\mathrm{R}}}{\pi}\left[\frac{y}{4}\left(\frac{1}{6} - \frac{1}{\pi^2}\right) + \frac{C_{\mathrm{A}}}{8\pi^2 g_{\mathrm{s}}^2}\right]\log(g_{3\mathrm{d}}^2 b_\perp)\,,$$

Short-distance behavior :

The kernel follows the same behavior as the LO one, where we determine $\hat{q}_0$ from the data :

$$\frac{C_{\mathrm{QCD}}}{g_{3\mathrm{d}}^2}(b_\perp) \xrightarrow{b_\perp \ll 1/m_{\mathrm{D}}} -\frac{C_{\mathrm{R}}}{8\pi}\frac{\zeta(3)}{\zeta(2)}\left(-\frac{1}{2g_{\mathrm{s}}^2} + \frac{3y}{2}\right)(g_{3\mathrm{d}}^2 b_\perp)^2 \log(g_{3\mathrm{d}}^2 b_\perp) + \frac{1}{4}\frac{\hat{q}_0}{g_{3\mathrm{d}}^6}(g_{3\mathrm{d}}^2 b_\perp)^2\,,$$

In-medium splitting rates

Using the broadening kernel at hand, one can compute in-medium splitting rates

- In the AMY approach:

$$\frac{d\Gamma_{ij}}{dz}(P,z) = \frac{\alpha_s P_{ij}(z)}{[2Pz(1-z)]^2} \int \frac{\mathrm{d}^2\mathbf{p}_\perp}{(2\pi)^2} \,\mathrm{Re}\left[2\mathbf{p}_\perp \cdot \mathbf{g}_{(z,P)}(\mathbf{p}_\perp)\right]$$

where $g_{(z,P)}(p_\perp)$ is solution to the integral equation :

$$\begin{aligned} 2\mathbf{p}_\perp = i\delta E(z,P,\mathbf{p}_\perp)\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) + \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2}\, \bar{C}(q_\perp) \\ \times \Big\{ C_1 \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - \mathbf{q}_\perp)\right] \\ + C_z \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - z\mathbf{q}_\perp)\right] \\ + C_{1-z} \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - (1-z)\mathbf{q}_\perp)\right] \Big\} \,. \end{aligned}$$

In-medium splitting rates

BH regime :

$$Pz(1-z) \ll \omega_{\rm BH} \sim T$$

Formation time is small and interference between scatterings can be neglected.

One solves the rate equations in opacity expansion in the number of elastic scatterings with medium.

Amounts to calculating a numerical integral.

LPM regime :

$$Pz(1-z) \gg \omega_{\rm BH} \sim T$$

Typical number of rescatterings within the formation time of bremsstrahlung can be large.

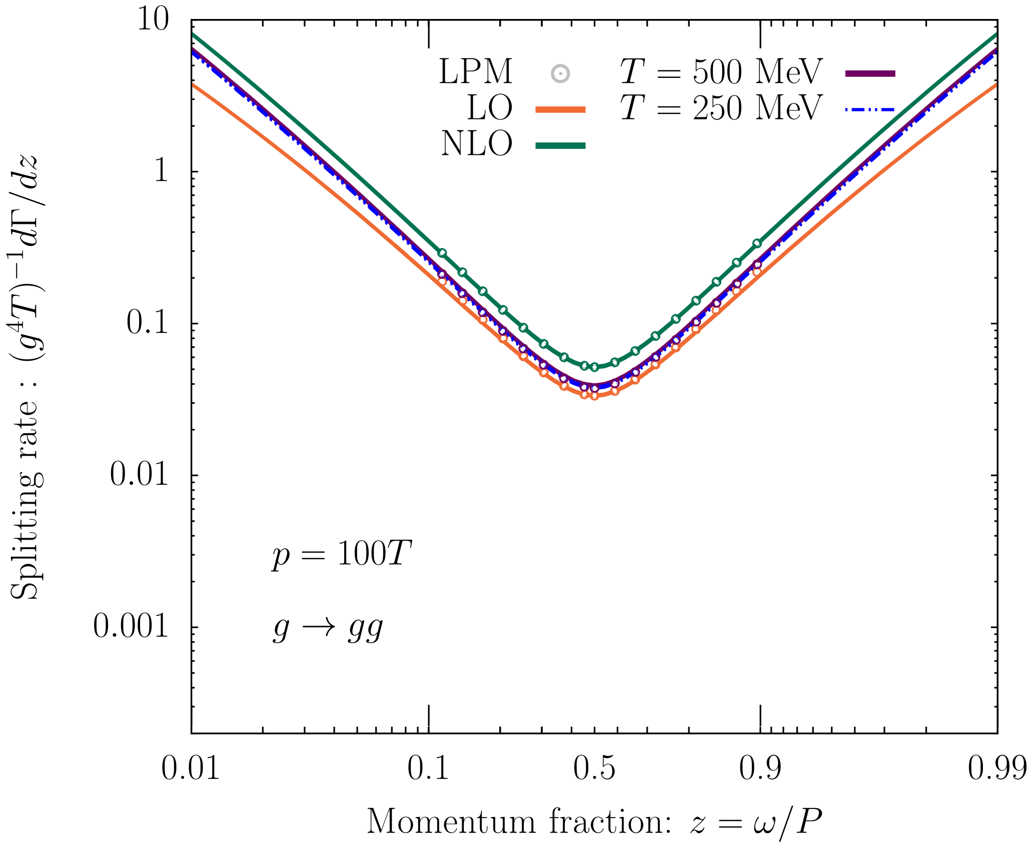
Interference of many soft scatterings need to be considered.

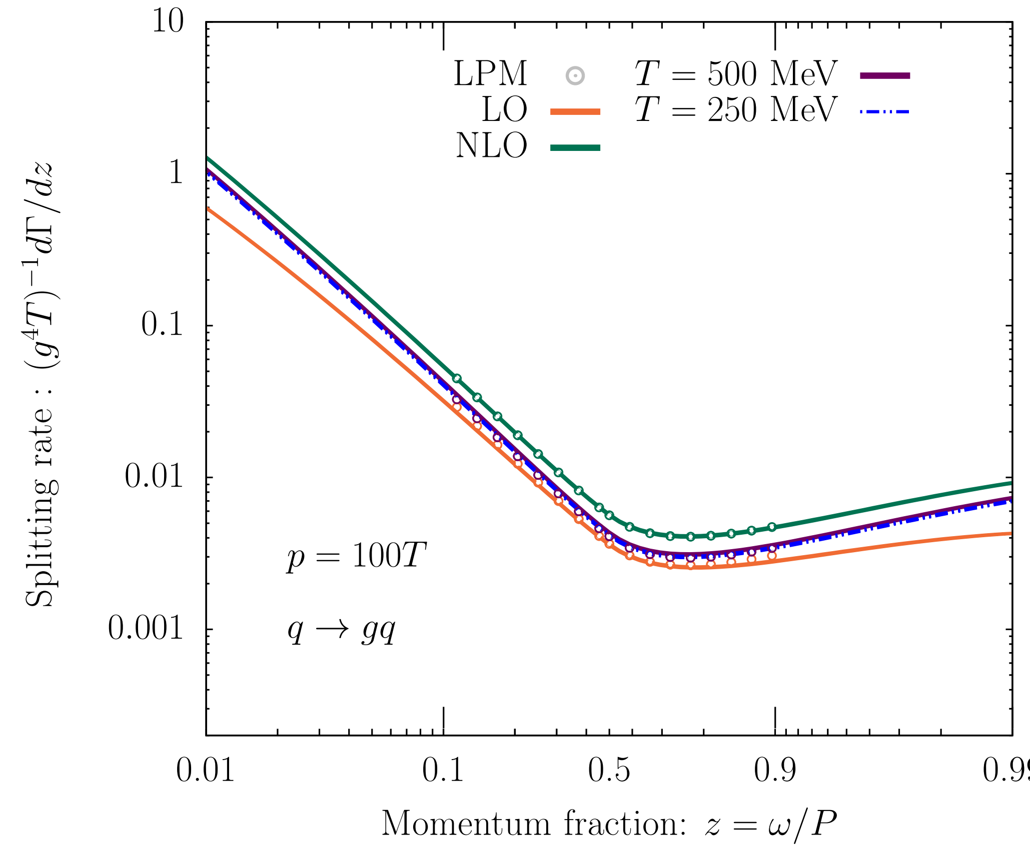
The broadening kernel follows a $b_\perp^2$ behavior at small distances

$$C(b_\perp) = -\frac{g_s^2 T^2}{16\pi} \mathcal{N} b_\perp^2 \log(\xi m_D^2 b_\perp^2/4)$$

The rate equations become a Harmonic oscillator problem and solved analytically

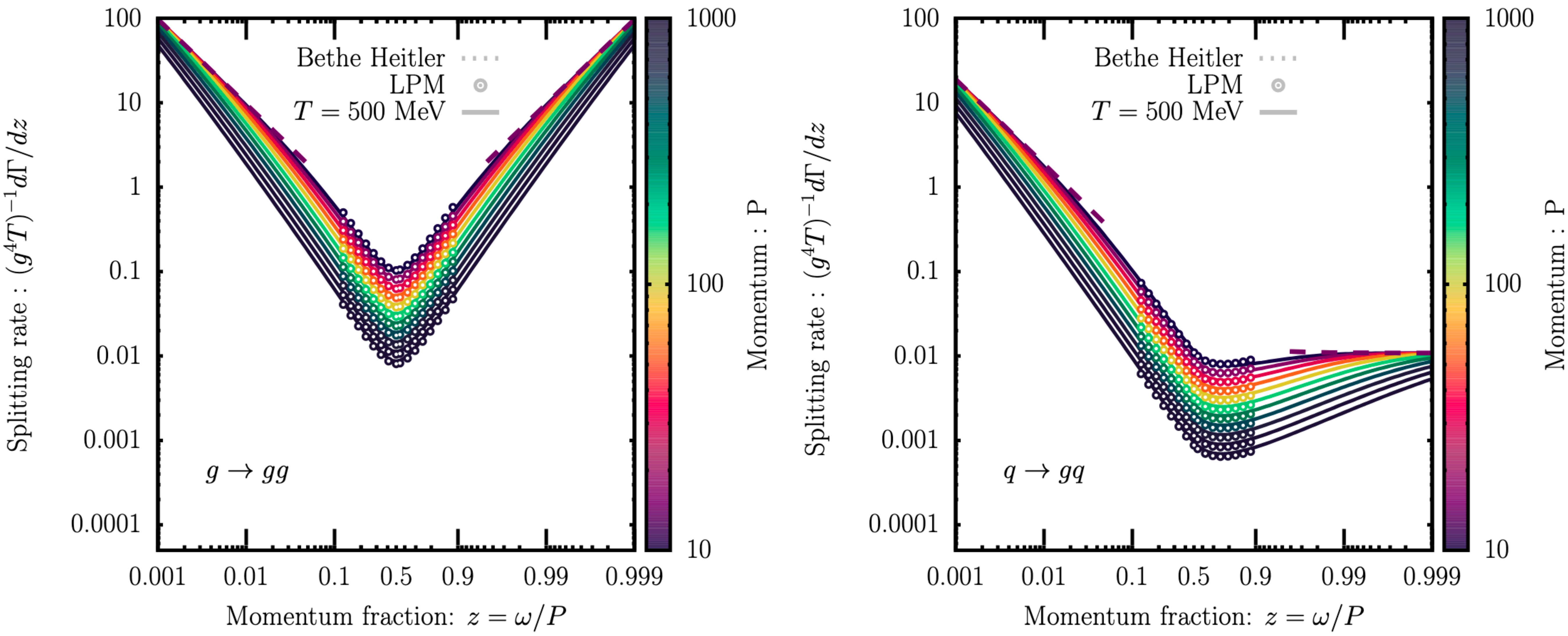
Results for non-perturbative splitting rates





- The two temperatures (250 MeV and 500 MeV) do not display a remarkable difference
- Recover LPM suppression at large momentum.
- In the large energy region the rate is closer to the LO one as they both follow the same behavior in broadening kernel at short distances
- In the small energy region the rate

Results for non-perturbative splitting rates



- BH regime at small energies

Finite medium

Preliminary results in finite medium

To compute the rate in finite medium it is best to work in momentum space

The rate follows the limits :

- $q \gg 1$:

$$C^{\mathrm{UV}}(q_\perp) = \frac{C_{\mathrm{R}}}{8\pi}\frac{\zeta(3)}{\zeta(2)}\left(-\frac{1}{2g_s^2}+\frac{3y}{2}\right)\frac{8\pi}{q_\perp^4}\,.$$

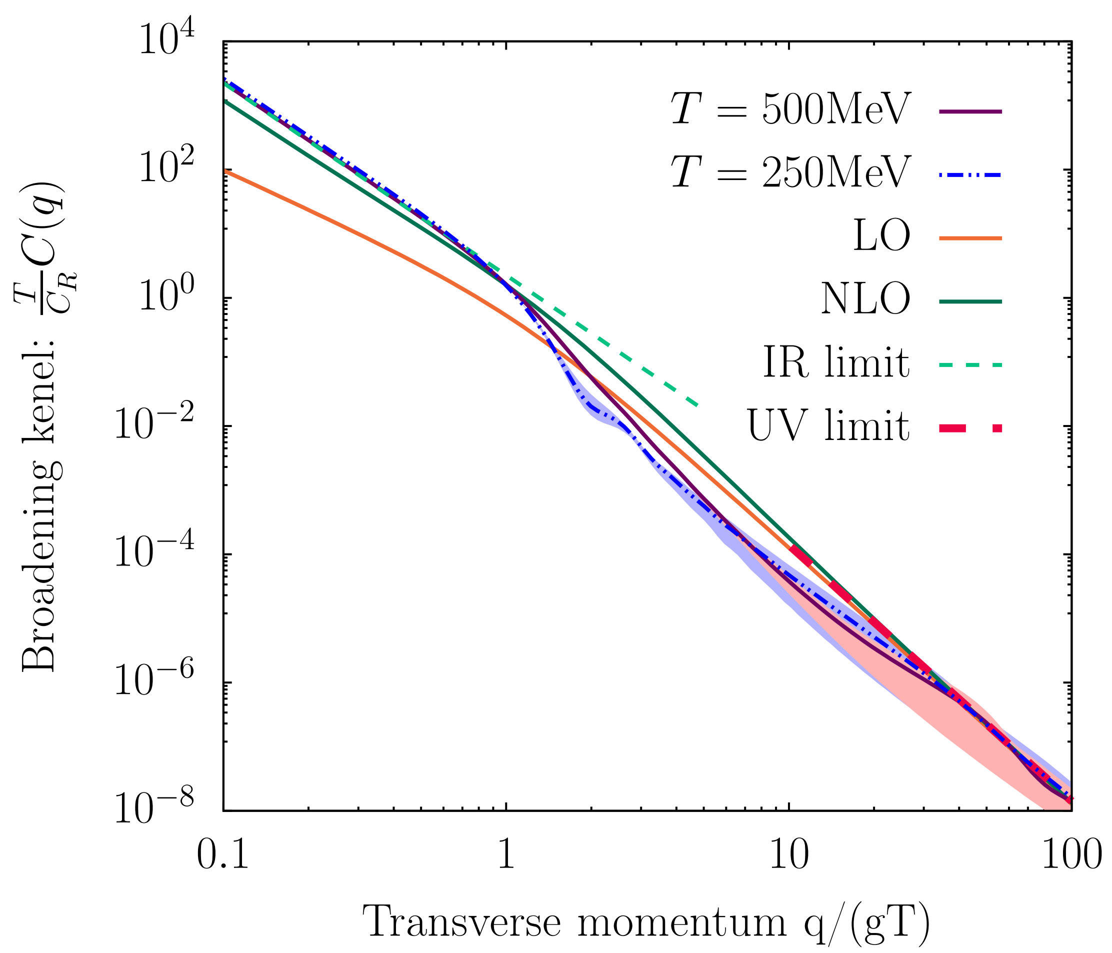
Similar to the LO rate

- $q \ll 1$:

$$C^{\mathrm{IR}}(q_\perp) = \frac{2\pi}{q_\perp^3}\frac{\sigma_{\mathrm{EQCD}}}{g_{3d}^2} + \frac{g^4 C_{\mathrm{R}}}{\pi}\left[\frac{y}{4}\left(\frac{1}{6}-\frac{1}{\pi^2}\right)+\frac{C_{\mathrm{A}}}{8\pi^2 g_s^2}\right]\frac{2\pi}{q_\perp^2}\,. \quad (6)$$

stems from the area law with string tension

Transverse Momentum Broadening:



Improved opacity expansion

- We compare the finite medium with an improved opacity expansion where after we cut low momentum interactions, we exponentiate higher order of the expansion

$$\frac{d\Gamma^a_{bc}}{dz} = \frac{g^2 P^a_{bc}(z)}{\pi P} \mathrm{Re} \int_{\boldsymbol{p}} \frac{1 - e^{-\left(i\delta E(\boldsymbol{p}) + \Sigma(\boldsymbol{p}^2)\right)t}}{i\delta E(\boldsymbol{p}) + \Sigma(\boldsymbol{p}^2)} \psi^{(1)}(\boldsymbol{p}) \,.$$

where the first order wave function is the collision integral of the initial condition

$$\psi^{(1)}(\boldsymbol{p}) = \int_{\boldsymbol{q}} n(t) C(\boldsymbol{q}) \left\{ C_1 \left[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{p^2 - \boldsymbol{p}\cdot\boldsymbol{q}}{\epsilon(\boldsymbol{p}-\boldsymbol{q})} \right] + C_z \left[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{p^2 + z\boldsymbol{p}\cdot\boldsymbol{q}}{\epsilon(\boldsymbol{p}+z\boldsymbol{q})} \right] + C_{1-z} \left[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{(p^2 + (1-z)\boldsymbol{p}\cdot\boldsymbol{q})}{\epsilon((\boldsymbol{p}+(1-z)\boldsymbol{q}))} \right] \right\}$$

and the subsequent interactions are encoded in the exponential of

$$\Sigma(M^2) \equiv \int_{\vec{k}^2 > M^2} n(t) C(\vec{k}) (C_1 + C_z + C_{1-z})$$

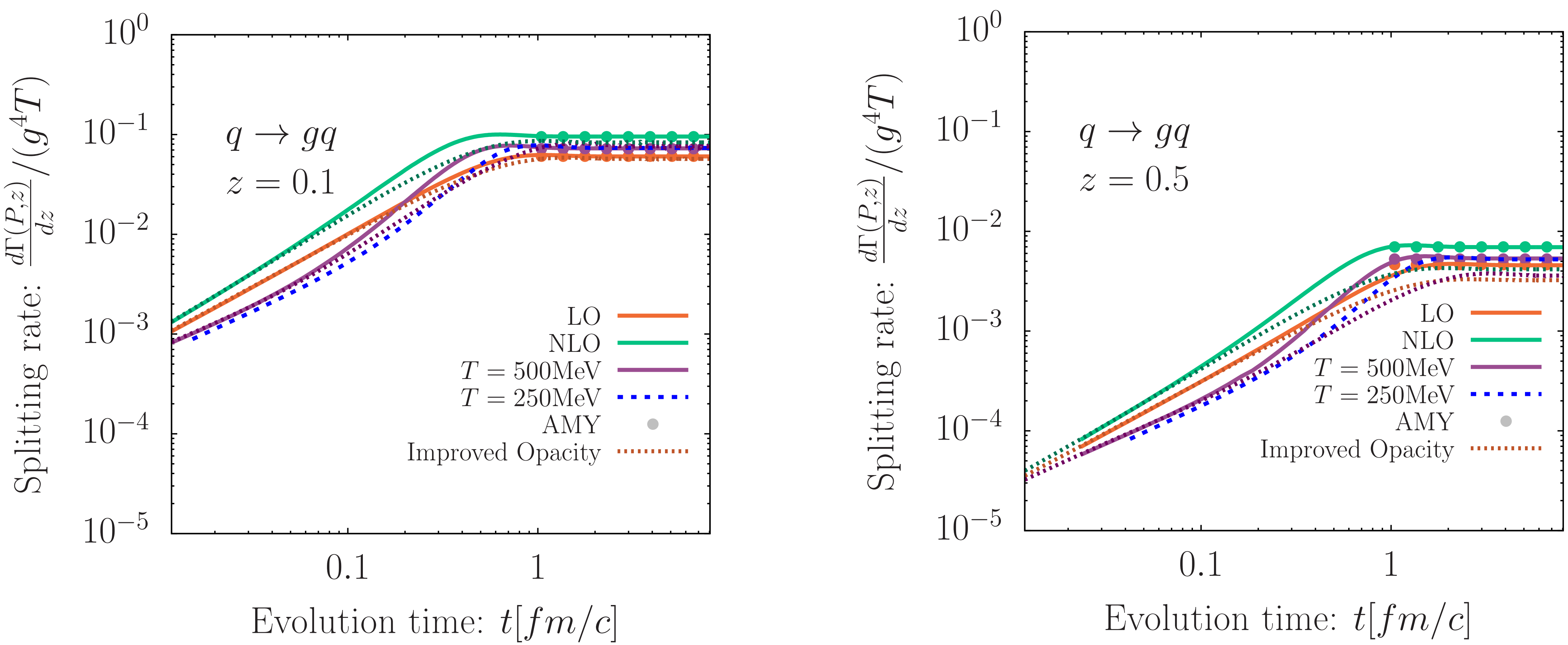
[C. Andres et Al. JHEP 03 (2021), 102]

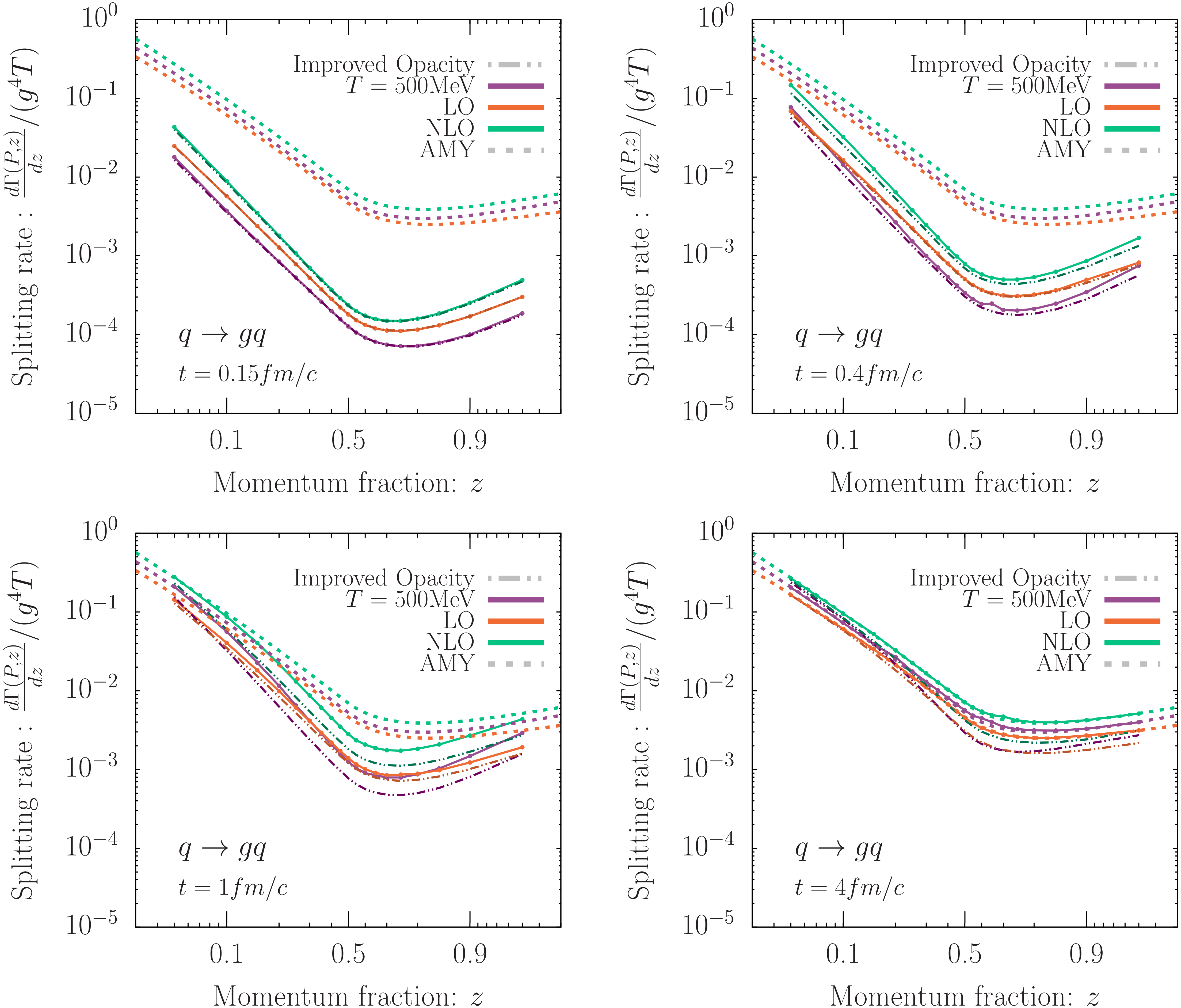
Preliminary results in finite medium

- We compute the rate in finite-medium following the approach of S. Caron-Huot et Al.

[S. Caron-Huot, C. Gale Phys.Rev.C 82 (2010), 064902]

Medium-induced splitting rates:




Improved Opacity
T = 500MeV
LO
NLO
AMY
q → gq
t = 0.15fm/c
t = 0.4fm/c
t = 1fm/c
t = 4fm/c
Splitting rate : dΓ(P,z)/dz /(g^4 T)
Momentum fraction: z
0.1
0.5
0.9
10^0
10^-1
10^-2
10^-3
10^-4
10^-5

Conclusion & Outlook

CRC-TR 211

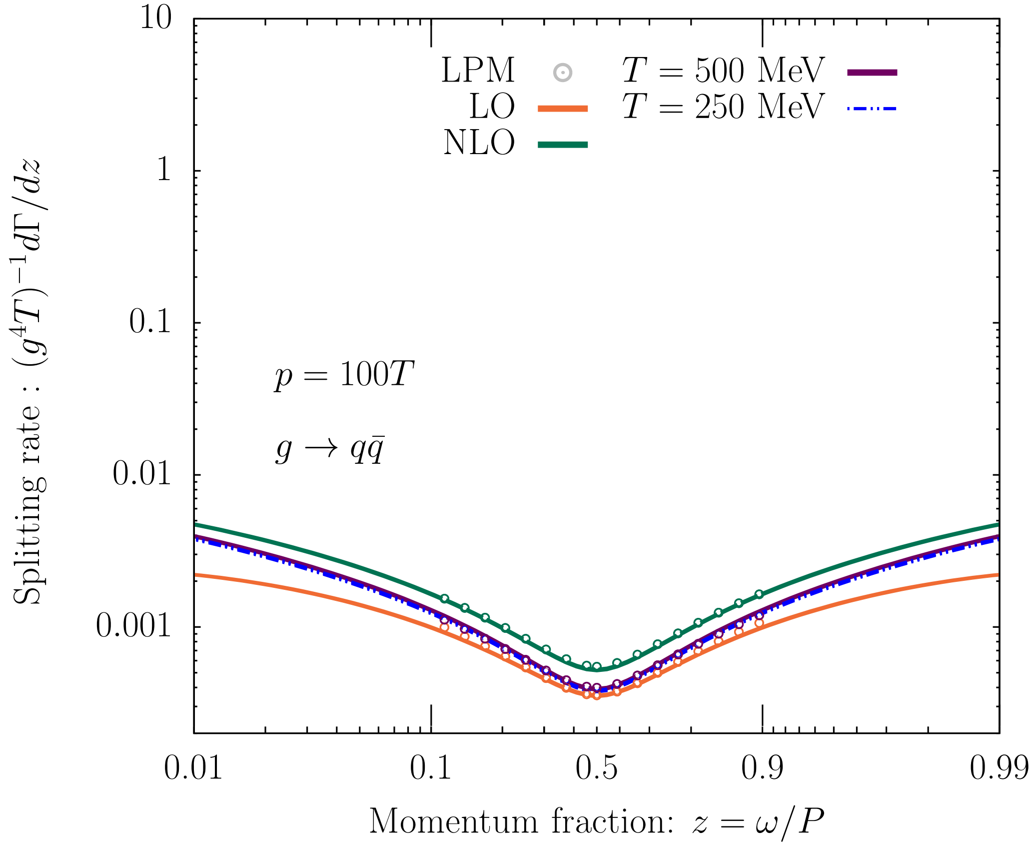
- We obtained fully matched lattice extracted non-perturbative contribution to momentum broadening in position space
- Using this kernel we computed non-perturbative contributions to the splitting rates in both infinite medium
- Successfully transformed the broadening kernel to momentum space which allows us to compute finite medium splittings

- Would be interesting to include the non-perturbative results to jet studies (elastic and radiative interactions) in kinetic studies or MC

Thank you!

Backup

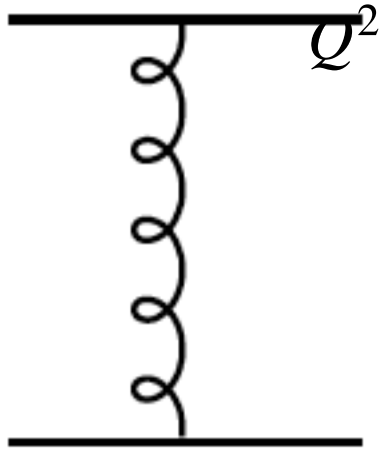
Gluon splitting



Non-perturbative splitting rates

CRC-TR 211

- Elastic interaction with the thermal bath are well described by perturbative theory at large $q_\perp \gg gT$
- But when we consider smaller $q_\perp \sim gT$ we require non-perturbative input.



$Q^2 = \omega^2 - q^2$

- For temperatures well above Tc, one can use Electrostatic QCD to compute the elastic broadening kernel.

[P. Arnold & W. Xiao Phys.Rev.D 78 (2008), 125008]

[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]