

# Non-perturbative contribution to collisional broadening and in-medium splittings

Ismail Soudi

Universität Bielefeld

Based on: G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv: 2105.01679  
S. Schlichting, I.S. work in progress

DPG Meeting of the Matter and Cosmos Section,  
September 2nd, 2021

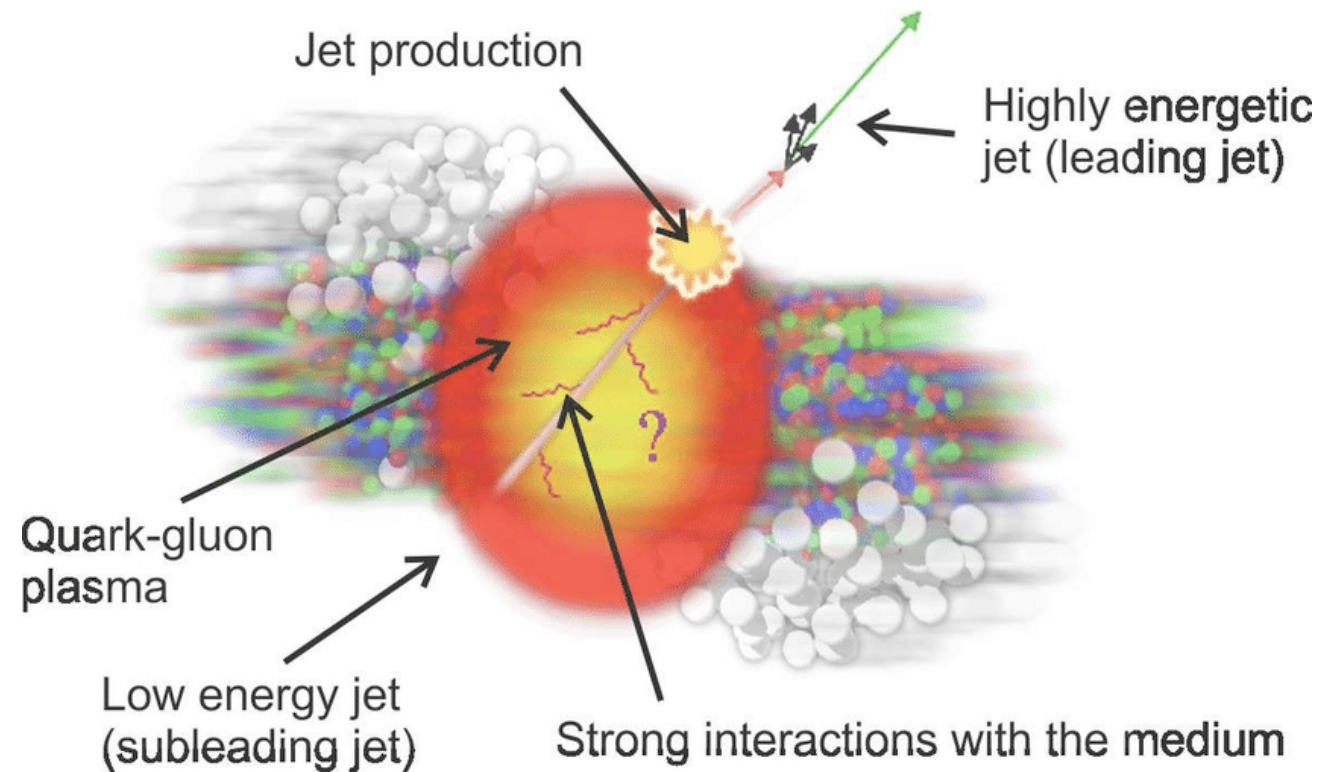


Bundesministerium  
für Bildung  
und Forschung



UNIVERSITÄT  
BIELEFELD

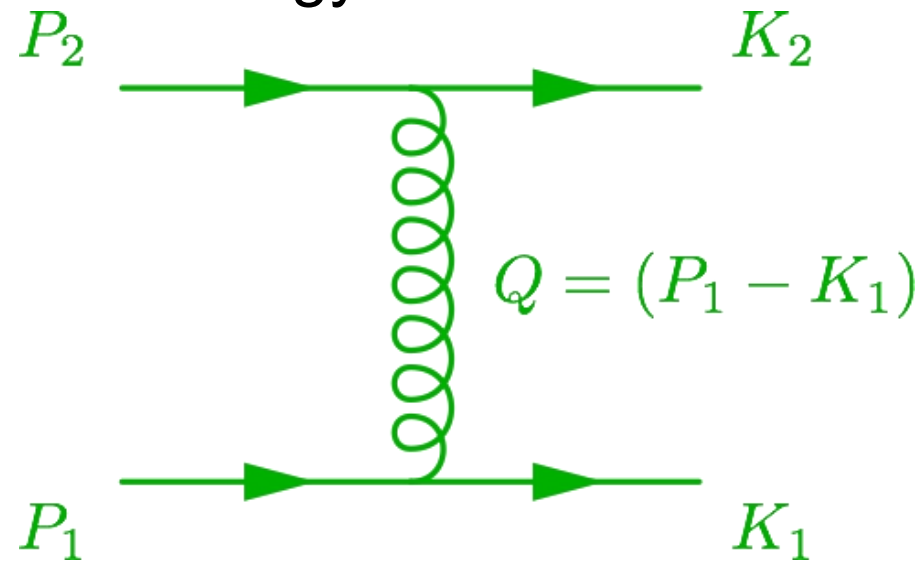




- Quark-Gluon-Plasma (QGP) is formed during Heavy-Ion collisions at RHIC and LHC
- Different probes of the QGP : collective flow, heavy flavor, EM probes..
- High energetic particles (jets) created at the early stage of the collision can be used as probes to understand the medium

As the high energetic partons traverse the medium they lose energy due to:

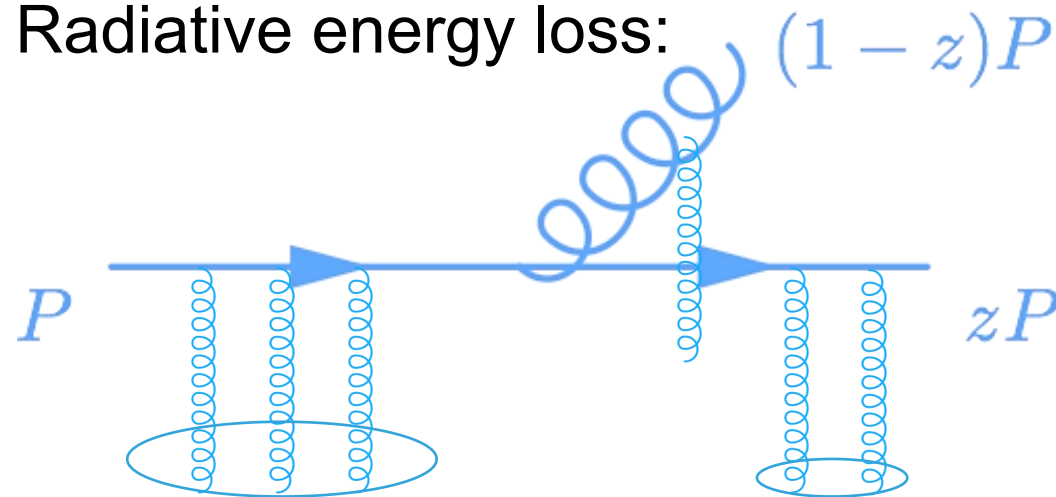
- Elastic energy loss:



Both require input from the medium  
transverse scattering rate :

$$\lim_{p \rightarrow \infty} \frac{d\Gamma(p, p + q_{\perp})}{d^2 q_{\perp}} = \frac{\mathcal{C}(q_{\perp})}{(2\pi)^2}.$$

- Radiative energy loss:



In the literature one employs pQCD broadening kernels:

- Static screened color centers  $\rightarrow C(q) \propto \frac{1}{(q^2 + m_D^2)^2}$
- Dynamics moving charges  $\rightarrow C(q) \propto \frac{1}{q^2(q^2 + m_D^2)}$
- Multiple soft scattering  $\rightarrow C(b) \propto \frac{q}{4} b^2$

[X. Wang and M. Gyulassy.  
*Phys.Rev.Lett.* 68 (1992) 1480-1483]

[P. Aurenche, F. Gelis,  
and H. Zaraket. *JHEP* 05  
(2002), p. 043.]

[Baier-Dokshitzer-  
Mueller-Peigne-Schiff]

Due to the infamous infrared problem of finite temperature QCD

=> perturbative calculations can receive large non-perturbative contribution even at small coupling.

$$C(\mathbf{b}_\perp) \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp}\right) C(q_\perp).$$

The collision kernel can be defined in terms of the behavior of certain light-like Wilson loops

$$C(\mathbf{b}_\perp) \equiv - \lim_{L \rightarrow \infty} \frac{1}{L} \ln \tilde{W}(L, \mathbf{b}_\perp),$$

[J. Casalderrey-Solana & D. Teaney, JHEP, vol. 04, p. 039, 2007]

=> For temperatures well above  $T_c$  these Wilson loops can be recast in the reduced effective theory of electrostatic QCD (EQCD)

[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

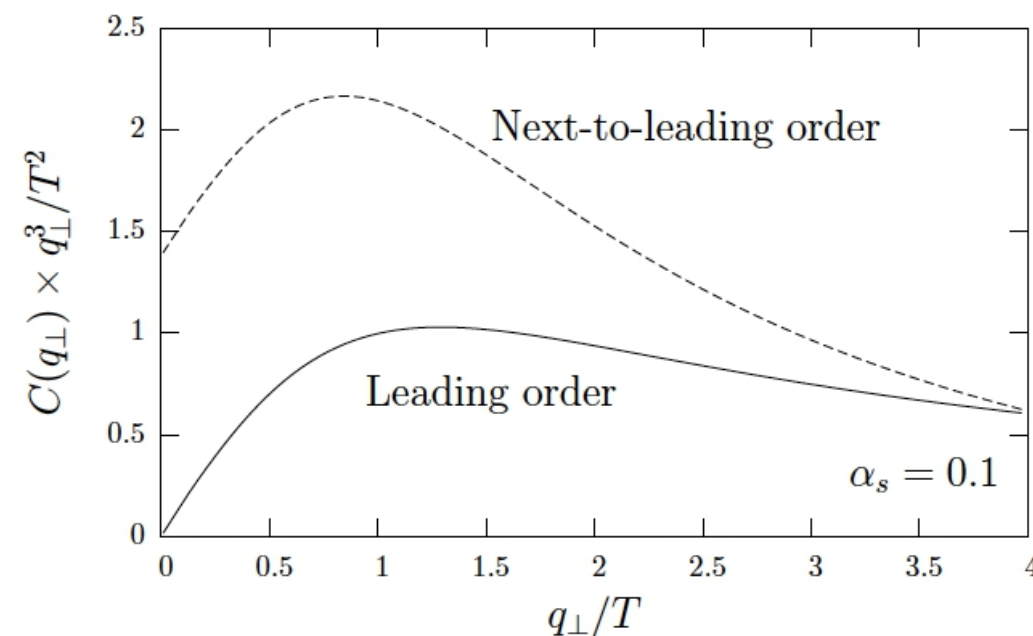
The kernel was computed in a perturbative expansion in the effective theory of EQCD :

- The LO EQCD kernel

$$C_{\text{QCD}}^{\text{LO}}(q_{\perp}) = \frac{g_s^4 T^3 C_R}{q_{\perp}^2 (q_{\perp}^2 + m_D^2)} \int \frac{d^3 p}{(2\pi)^3} \frac{p - p_z}{p} \left[ 2C_A n_B(p) (1 + n_B(p')) + 4N_f T_f n_F(p) (1 - n_F(p')) \right] = g_s^2 T C_R \begin{cases} \frac{m_D^2 - g_s^2 T^2 C_A \frac{q_{\perp}}{16T}}{q_{\perp}^2 (q_{\perp}^2 + m_D^2)}, & q_{\perp} \ll g_s T, \\ \frac{g_s^2 T}{q_{\perp}^4} \mathcal{N}, & q_{\perp} \gg g_s T, \end{cases}$$

[P. Arnold & W. Xiao Phys.Rev.D 78 (2008), 125008]

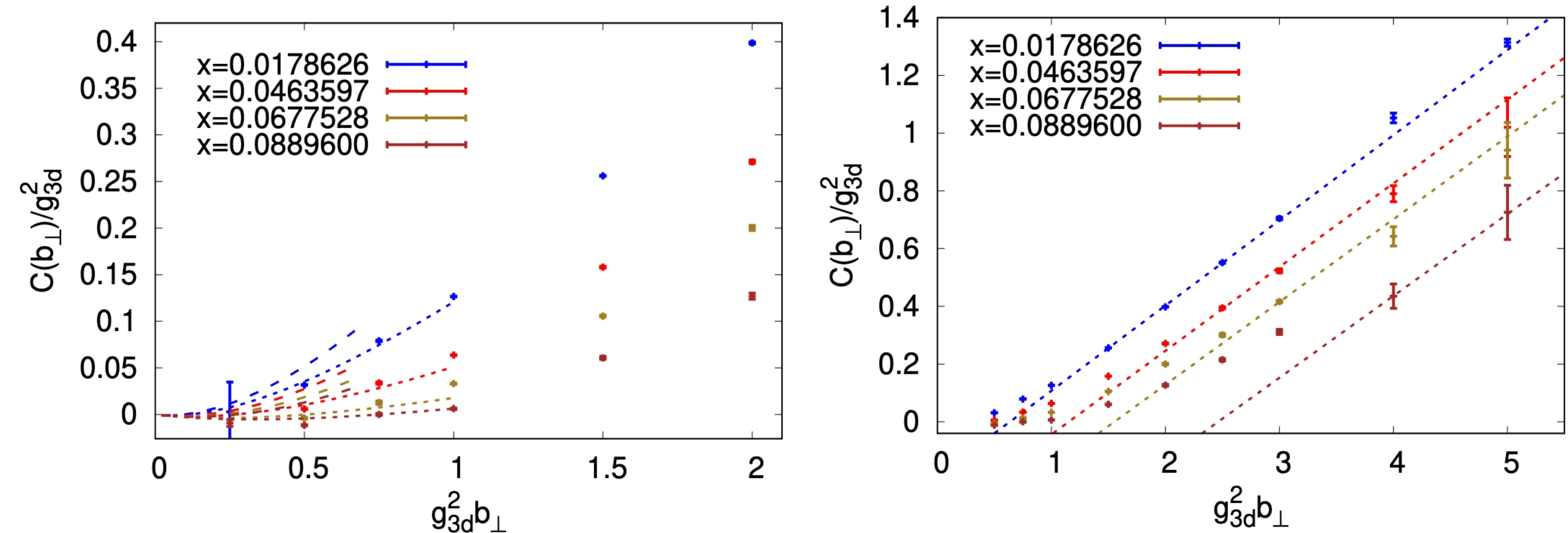
- NLO corrections :



[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

Beyond the perturbative result, lattice extracted non-perturbative contribution were computed

[G.D. Moore & N. Schlusser Phys.Rev.D 101 (2020) 1, 014505]



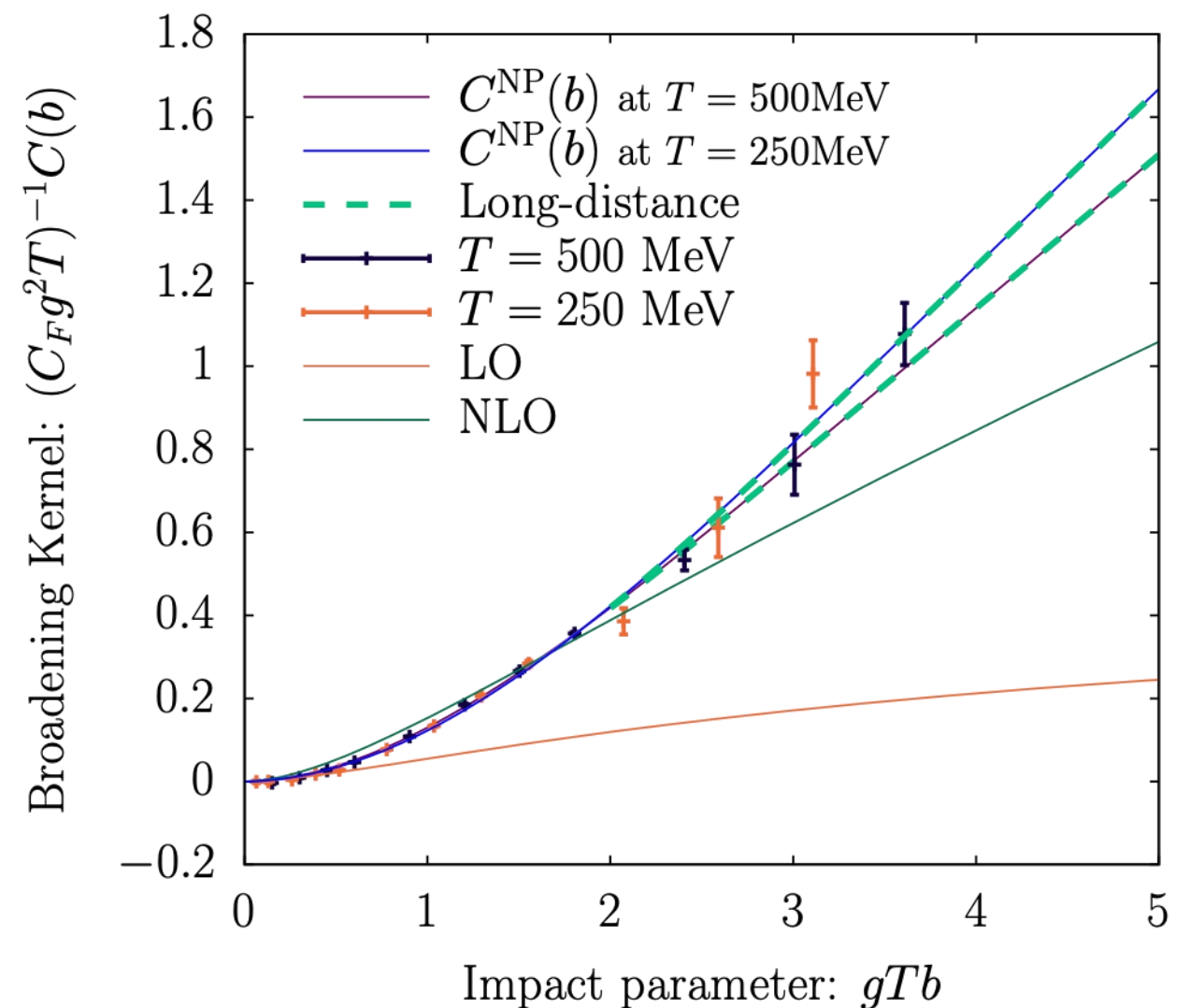
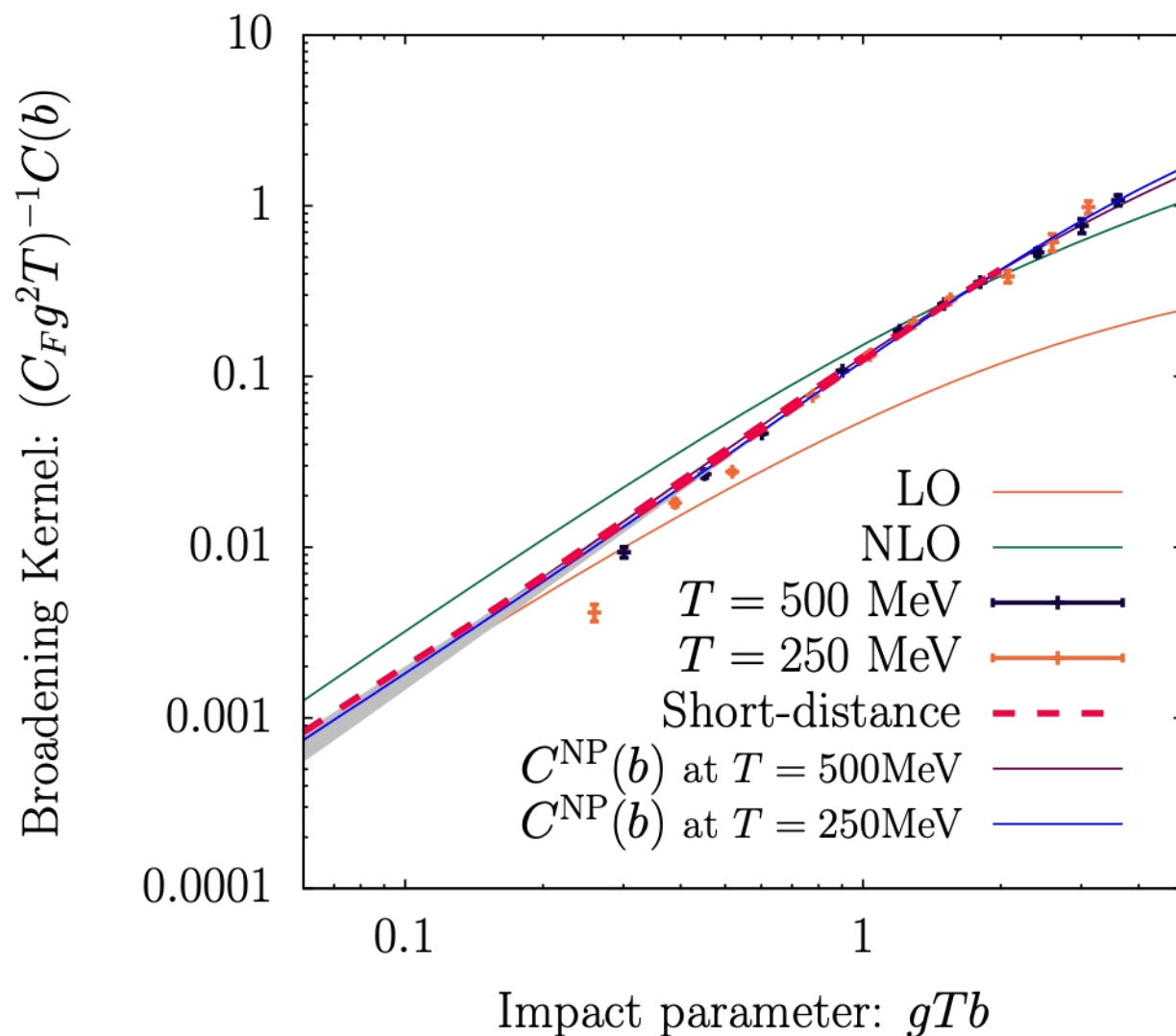
This result here is for the broadening kernel in EQCD which need to be matched to QCD



Since EQCD is a low-energy effective theory for QCD they should both agree in the IR regime but in the UV they can be different.

In order to ensure the right UV behavior while keeping the IR behavior from the lattice result we write the full kernel:

$$C_{\text{QCD}}(b_{\perp}) \approx \left( C_{\text{QCD}}^{\text{pert}}(b_{\perp}) - C_{\text{EQCD}}^{\text{pert}}(b_{\perp}) \right) + C_{\text{EQCD}}^{\text{latt}}(b_{\perp}).$$



[G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv:2105.01679]



Long-distance behavior :

The kernel follows an area-law with sub-leading logarithm corrections

[M. Laine, Eur. Phys. J. C, vol. 72]

$$\frac{C_{\text{QCD}}}{g_{3\text{d}}^2}(b_{\perp}) \xrightarrow{b_{\perp} \gg 1/g_{3\text{d}}^2} \boxed{A + \frac{\sigma_{\text{EQCD}}}{g_{3\text{d}}^4} g_{3\text{d}}^2 b_{\perp}} + \frac{g_s^4 C_{\text{R}}}{\pi} \left[ \frac{y}{4} \left( \frac{1}{6} - \frac{1}{\pi^2} \right) + \frac{C_{\text{A}}}{8\pi^2 g_s^2} \right] \log(g_{3\text{d}}^2 b_{\perp}),$$

Short-distance behavior :

The kernel follows the same behavior as the LO one, where we determine  $\hat{q}_0$  from the data :

$$\frac{C_{\text{QCD}}}{g_{3\text{d}}^2}(b_{\perp}) \xrightarrow{b_{\perp} \ll 1/m_{\text{D}}} -\frac{C_{\text{R}}}{8\pi} \frac{\zeta(3)}{\zeta(2)} \left( -\frac{1}{2g_s^2} + \frac{3y}{2} \right) (g_{3\text{d}}^2 b_{\perp})^2 \log(g_{3\text{d}}^2 b_{\perp}) + \frac{1}{4} \frac{\hat{q}_0}{g_{3\text{d}}^6} (g_{3\text{d}}^2 b_{\perp})^2,$$

Using the broadening kernel at hand, one can compute in-medium splitting rates

- In the AMY approach:

[Arnold-Moore-Yaffe]

$$\frac{d\Gamma_{ij}}{dz}(P, z) = \frac{\alpha_s P_{ij}(z)}{[2Pz(1-z)]^2} \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \text{Re} [2\mathbf{p}_\perp \cdot \mathbf{g}_{(z,P)}(\mathbf{p}_\perp)]$$

where  $\mathbf{g}_{(z,P)}(\mathbf{p}_\perp)$  is solution to the integral equation :

$$\begin{aligned} 2\mathbf{p}_\perp = i\delta E(z, P, \mathbf{p}_\perp) \mathbf{g}_{(z,P)}(\mathbf{p}_\perp) &+ \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} \bar{C}(q_\perp) \\ &\times \left\{ C_1 [\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - \mathbf{q}_\perp)] \right. \\ &\quad + C_z [\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - z\mathbf{q}_\perp)] \\ &\quad \left. + C_{1-z} [\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - (1-z)\mathbf{q}_\perp)] \right\} . \end{aligned}$$

BH regime :

$$Pz(1-z) \ll \omega_{\text{BH}} \sim T$$

Formation time is small and interference between scatterings can be neglected.

One solves the rate equations in opacity expansion in the number of elastic scatterings with medium.

Amounts to calculating a numerical integral.

[Gyulassy-Levai-Vitev]

LPM regime :

$$Pz(1-z) \gg \omega_{\text{BH}} \sim T$$

Typical number of rescatterings within the formation time of bremsstrahlung can be large.

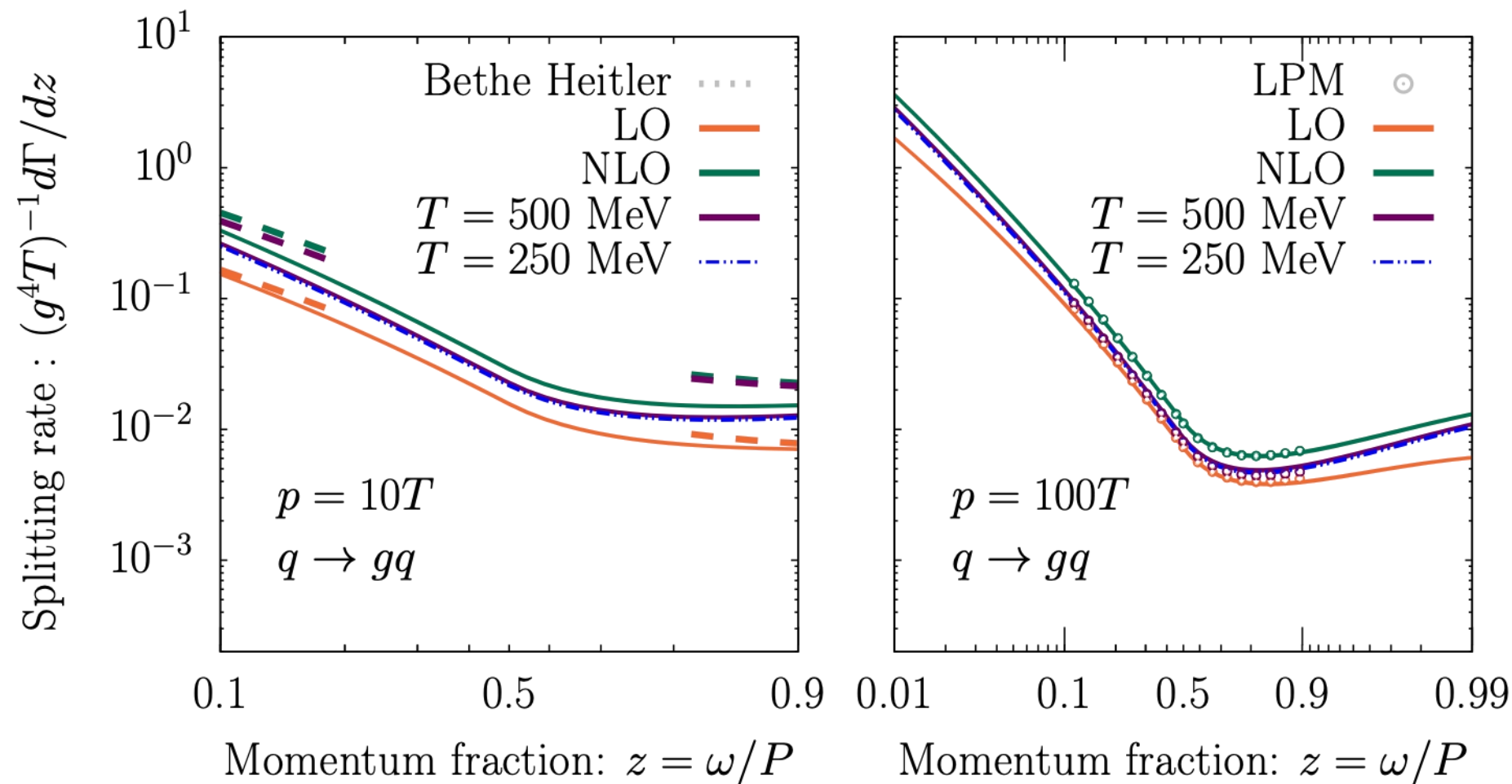
Interference of many soft scatterings need to be considered.

The broadening kernel follows a  $b_{\perp}^2$  behavior at small distances

$$C(b_{\perp}) = -\frac{g_s^2 T^2}{16\pi} \mathcal{N} b_{\perp}^2 \log(\xi m_D^2 b_{\perp}^2 / 4)$$

The rate equations become a Harmonic oscillator problem and solved analytically

[P. Arnold and C. Dogan.  
*Phys. Rev. D* 78 (2008), p.  
065008 ]



- The two temperatures (250 MeV and 500 MeV) do not display a remarkable difference
- Recover LPM suppression at large momentum.
- In the large energy region the rate is closer to the LO one as they both follow the same behavior in broadening kernel at short distances

Finite medium

To compute the rate in finite medium it is best to work in momentum space

The rate follows the limits :

- $q \gg 1$ :

$$C^{\text{UV}}(q_{\perp}) = \frac{C_R}{8\pi} \frac{\zeta(3)}{\zeta(2)} \left( -\frac{1}{2g_s^2} + \frac{3y}{2} \right) \frac{8\pi}{q_{\perp}^4}.$$

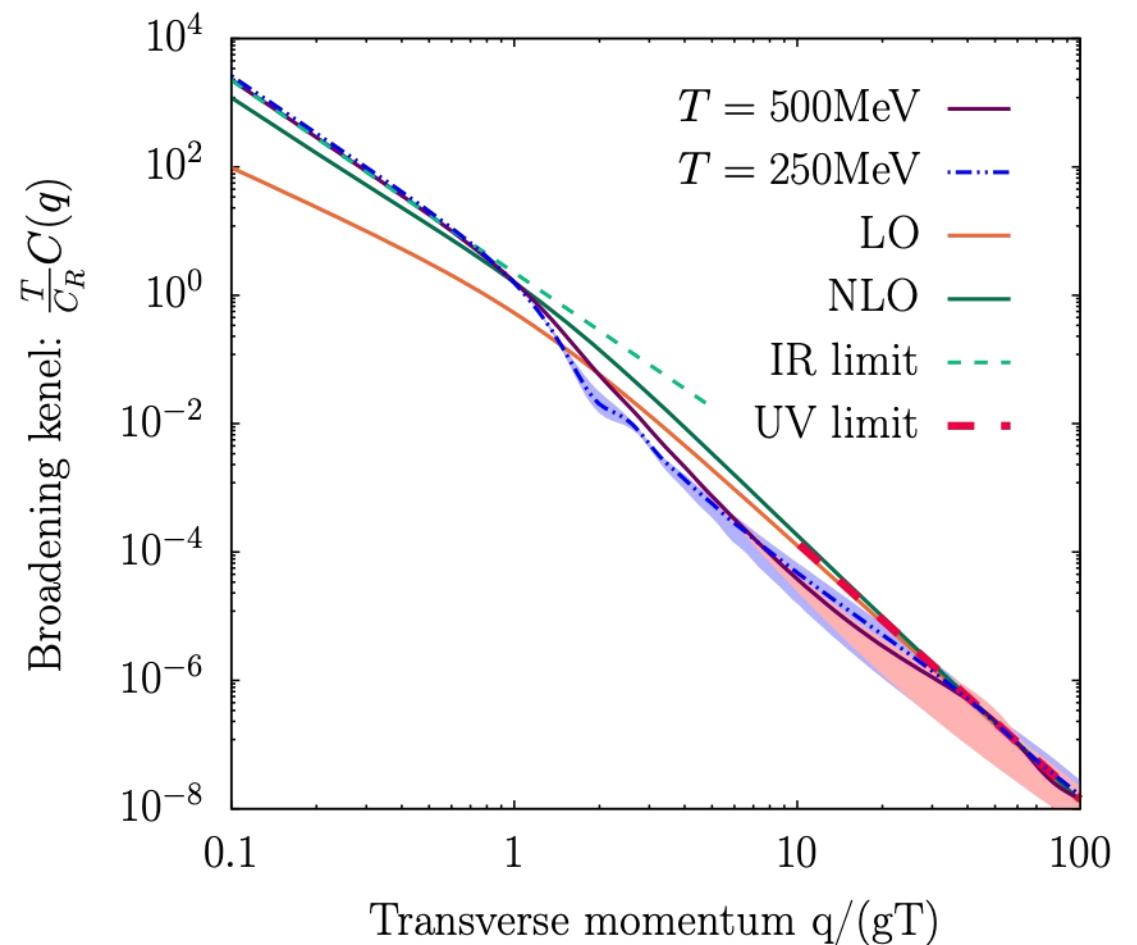
Similar to the LO rate

- $q \ll 1$ :

$$C^{\text{IR}}(q_{\perp}) = \frac{2\pi}{q_{\perp}^3} \frac{\sigma_{\text{EQCD}}}{g_{3d}^2} + \frac{g^4 C_R}{\pi} \left[ \frac{y}{4} \left( \frac{1}{6} - \frac{1}{\pi^2} \right) + \frac{C_A}{8\pi^2 g_s^2} \right] \frac{2\pi}{q_{\perp}^2}. \quad (6)$$

stems from the area law with string tension

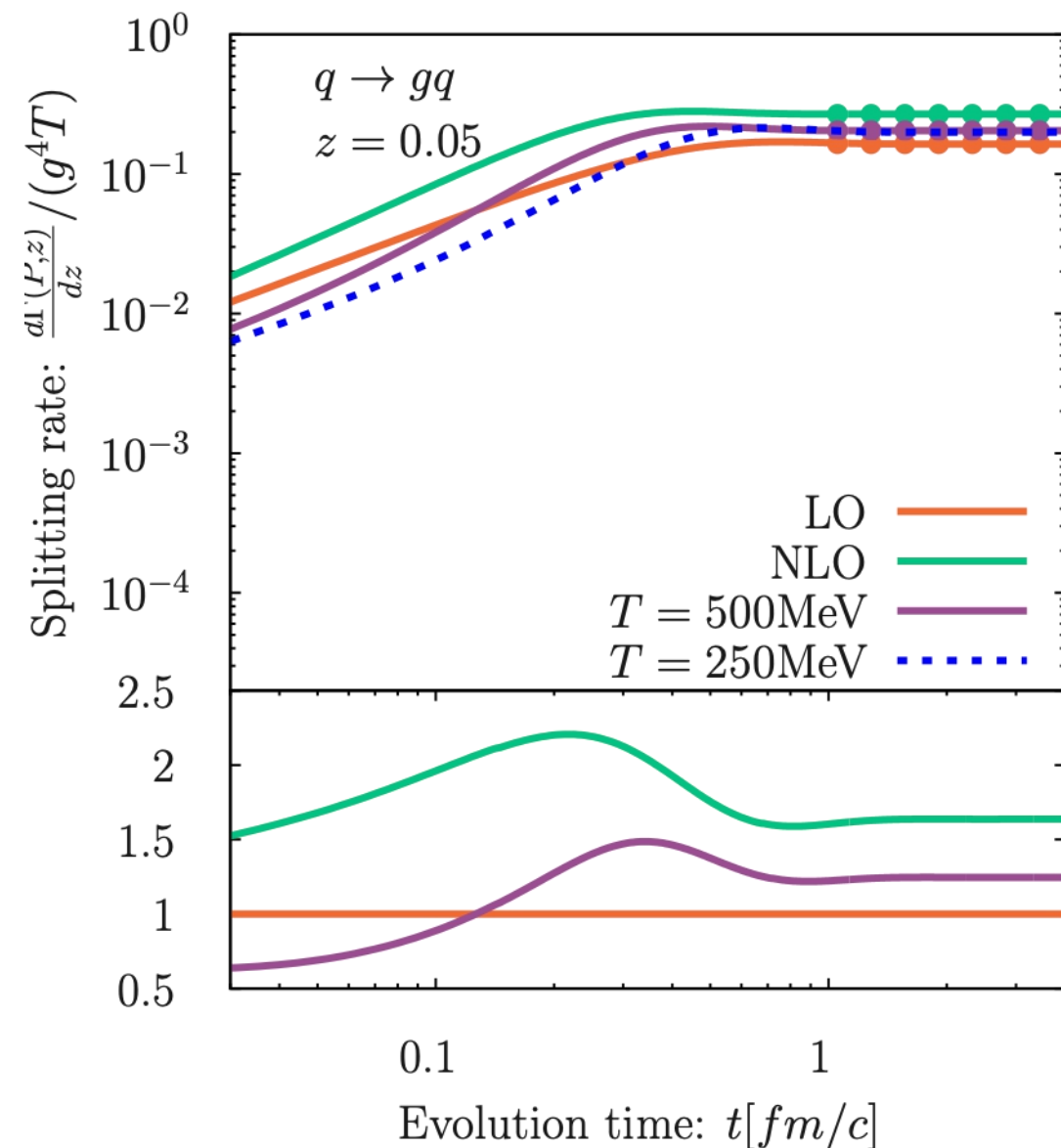
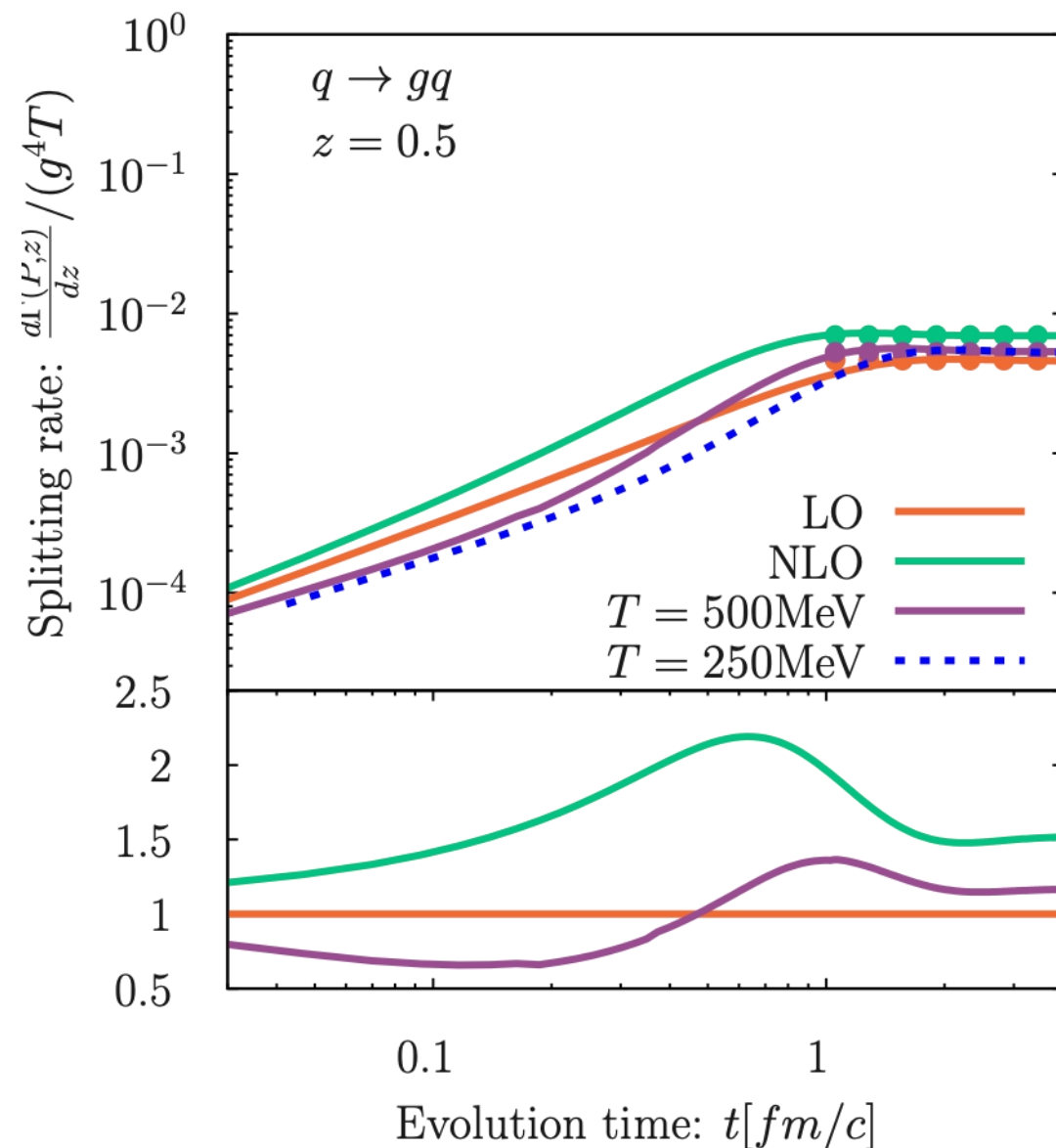
Transverse Momentum Broadening:



- We compute the rate in finite-medium following the approach of S. Caron-Huot and C. Gale

[S. Caron-Huot, C. Gale Phys.Rev.C 82 (2010), 064902]

Medium-induced splitting rates:  $P = 300T$





- We compare the finite medium with an improved opacity expansion where after we cut low momentum interactions, we exponentiate higher order of the expansion

$$\left. \frac{d\Gamma_{bc}^a}{dz} \right|_{N=X} (P, z, \tilde{t}) = \frac{g^4 T P_{bc}^a(z)}{\pi} \text{Re} \int_0^{\tilde{t}} d\Delta \tilde{t} \int_{\tilde{p}} e^{-(i\delta \tilde{E}(\tilde{p}) + \Lambda \Sigma_3(\tilde{p}^2))\Delta \tilde{t}} \tilde{\psi}_I^{(1)}(\tilde{p}) ,$$

[C. Andres et Al. JHEP 03 (2021), 102]

where the first order wave function is the collision integral of the initial condition

$$\begin{aligned} \psi^{(1)}(\mathbf{p}) = \int_{\mathbf{q}} n(t) C(\mathbf{q}) \left\{ C_1 \left[ \frac{p^2}{\epsilon(\mathbf{p})} - \frac{p^2 - \mathbf{p} \cdot \mathbf{q}}{\epsilon(\mathbf{p} - \mathbf{q})} \right] \right. \\ \left. + C_z \left[ \frac{p^2}{\epsilon(\mathbf{p})} - \frac{p^2 + z \mathbf{p} \cdot \mathbf{q}}{\epsilon(\mathbf{p} + z \mathbf{q})} \right] + C_{1-z} \left[ \frac{p^2}{\epsilon(\mathbf{p})} - \frac{p^2 + (1-z) \mathbf{p} \cdot \mathbf{q}}{\epsilon(\mathbf{p} + (1-z) \mathbf{q})} \right] \right\} \end{aligned}$$

and the subsequent interactions

are encoded in the exponential of  $\Sigma(M^2) \equiv \int_{k^2 > M^2} n(t) C(k) (C_1 + C_z + C_{1-z})$

- Improved opacity expansion around the Harmonic Oscillator

[Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk]

$$C(b_{\perp}) = \frac{g_s^4 T^3}{16\pi} \mathcal{N} b_{\perp}^2 \ln \left( \frac{4Q^2}{\xi m_D^2} \right) + \frac{g_s^4 T^3}{16\pi} \mathcal{N} b_{\perp}^2 \ln \left( \frac{1}{Q^2 b_{\perp}^2} \right) = C^{HO}(b_{\perp}) + C^{\text{pert}}(b_{\perp}) ,$$

$$\frac{dI^{NLO}}{dz}(P, z, t) = \frac{dI^{HO}}{dz}(P, z, t) + \frac{dI^{(1)}}{dz}(P, z, t) .$$

- By self-consistently solving for the scale

$$Q^2(P, z) = \sqrt{Pz(1-z)\hat{q}_3(Q^2)} ,$$

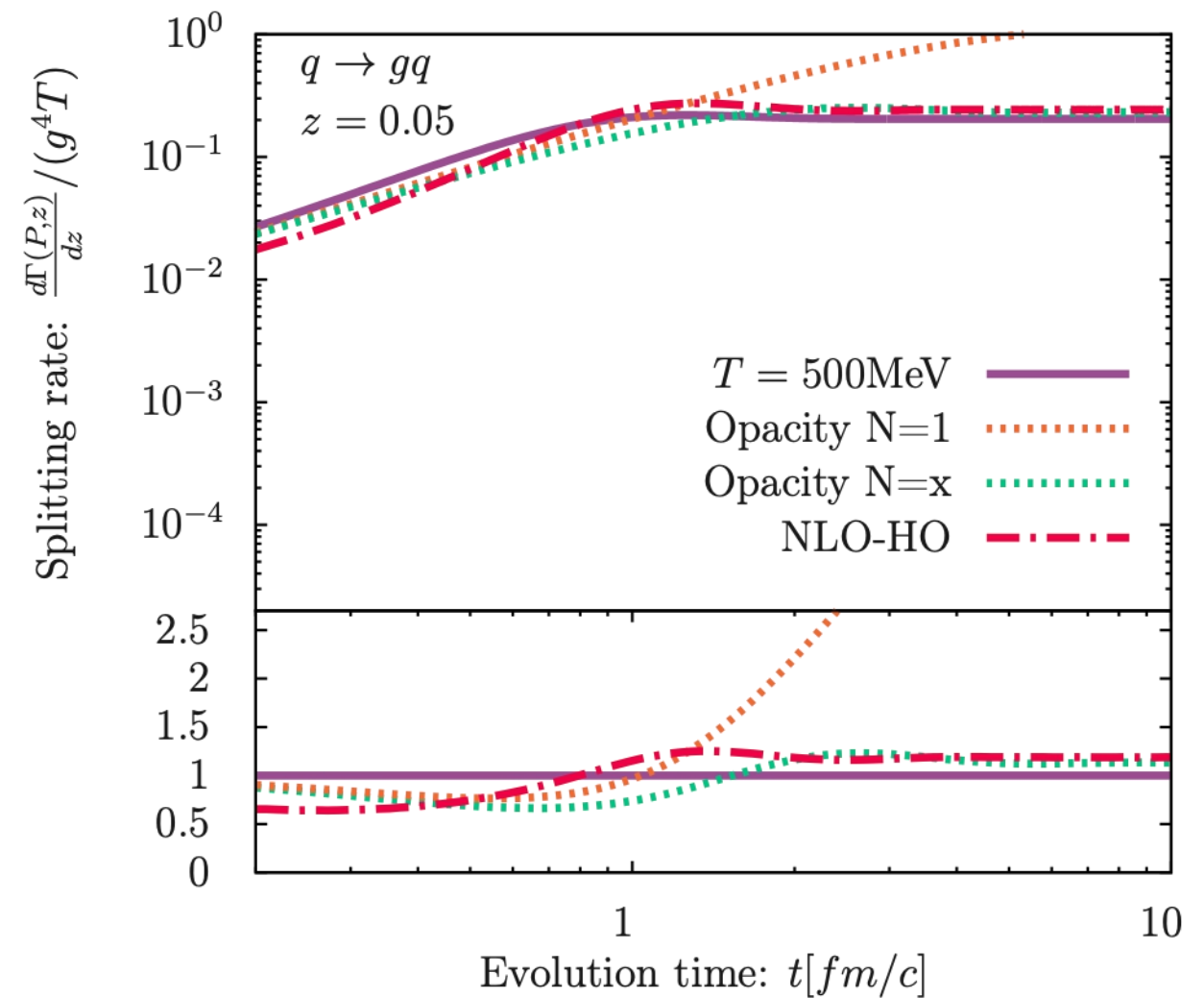
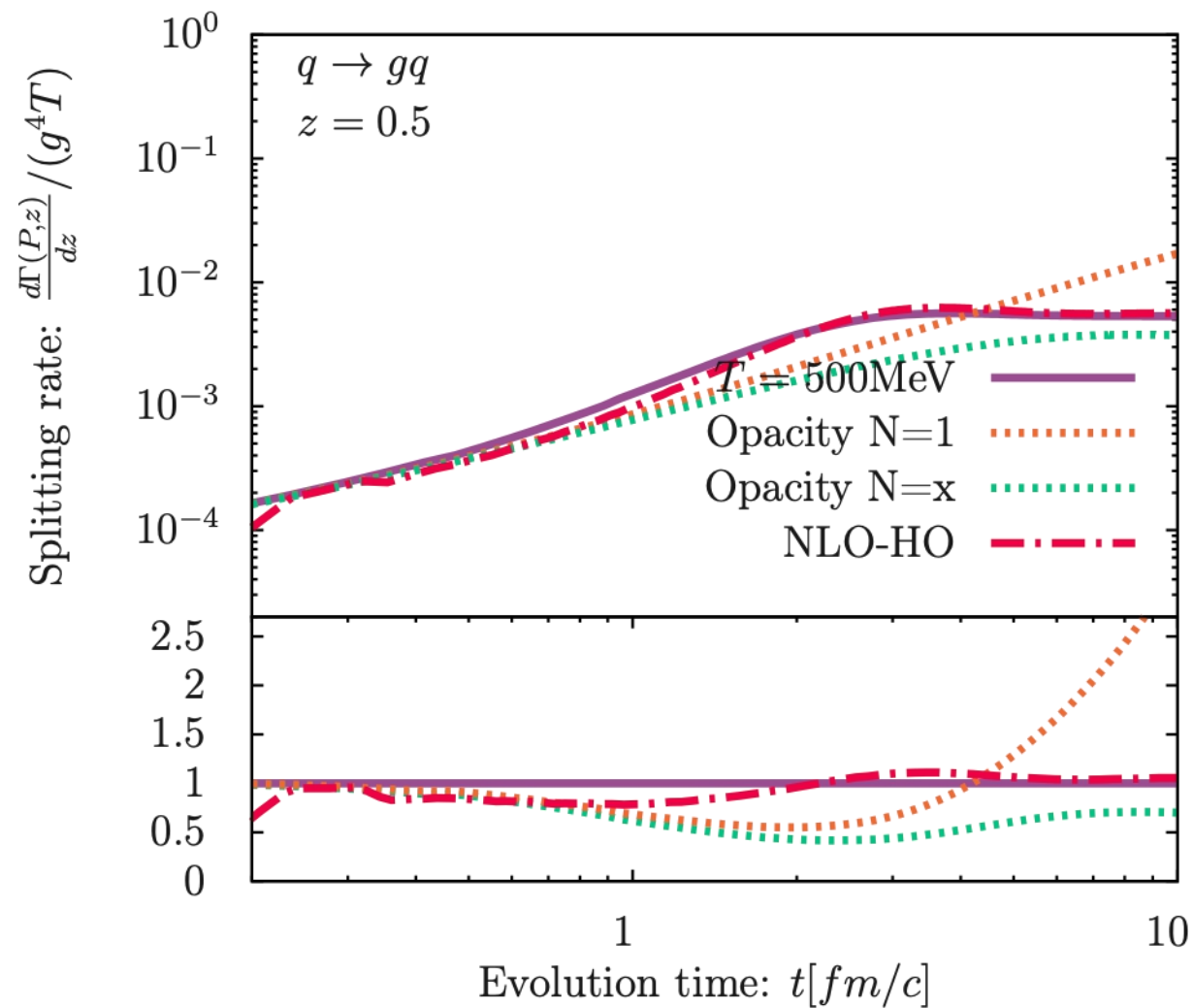
$$\hat{q}_{\text{eff}}(Q^2) = \frac{g_s^4 T^3}{4\pi} \mathcal{N} [C_1 + C_z z^2 + C_{1-z}(1-z)^2] \ln \left( \frac{4Q^2}{\xi m_D^2} \right) ,$$

- One can find correction to the HO

$$\frac{dI^{HO}}{dz}(P, z, t) = \frac{g^2}{4\pi^2} \ln |\cos \Omega t| ,$$

$$\frac{dI^{(1)}}{dz}(P, z, t) = \frac{g^2}{4\pi^2} \text{Re} \int_0^t ds \int_0^\infty \frac{2du}{u} [C_1 C^{(1)}(u) + C_z C^{(1)}(zu) + C_{1-z} C^{(1)}((1-z)u)] e^{k^2(s)u^2} ,$$

Medium-induced splitting rates:  $P = 300T$

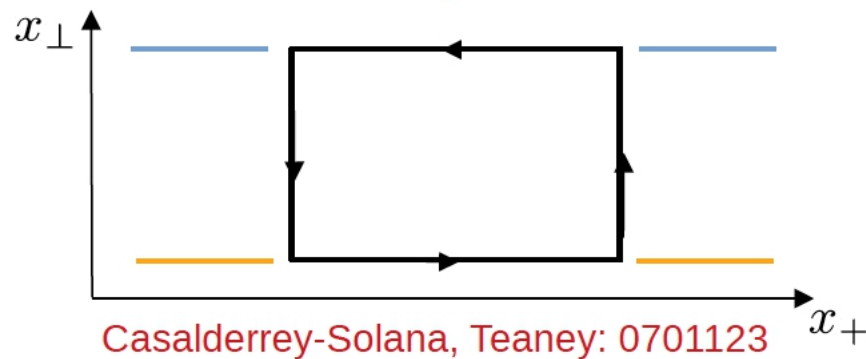


[G.D. Moore & N. Schlusser Phys.Rev.D 102 (2020) 9, 094512]

- Collision kernel

$$C(q_{\perp}) = \frac{d\Gamma}{d^2q_{\perp} dL}$$

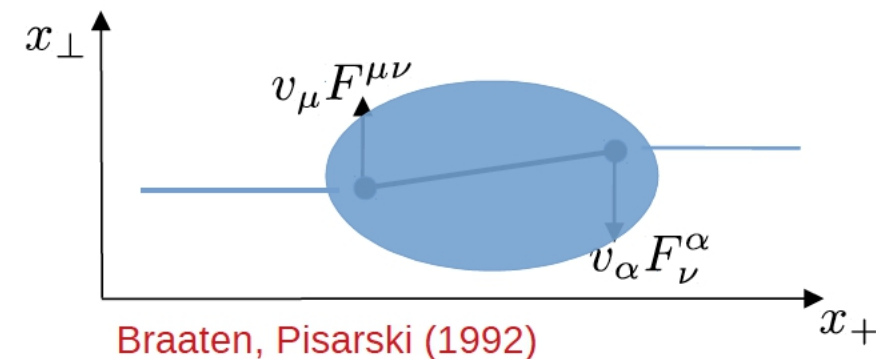
- Wilson loop



- Asymptotic mass

$$m_{\infty}^2 = C_R (Z_g + Z_f)$$

- Force-force-correlator



Nonperturbative gluon-zero-mode contributions:

→ calculate in lattice EQCD

Caron-Huot: 0811.1603

Slide courtesy of N. Schlusser

- Using non-perturbative contribution, we observe a sizable difference to the perturbative broadening kernel as well as to the in-medium splitting rates ( $\sim 25\%$ )
- Although the limiting behaviors are recovered in the limiting cases, a robust full calculation of the rate is important.

---

[G.D. Moore & N. Schlusser Phys.Rev.D 102 (2020) 9, 094512]

- There have been effort to extract asymptotic masses using the same procedure which still needs to be matched to QCD.
- Would be interesting to include the non-perturbative results to jet studies (elastic and radiative interactions) in kinetic studies or MC

Thank you!

Backup

# Results for non-perturbative splitting rates

