Non-perturbative contribution to collisional broadening and in-medium splittings

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Based on: G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv: 2105.01679
S. Schlichting, I.S. work in progress

Probing QCD at High Energy and Density with Jets,
August 11th, 2021

DFG

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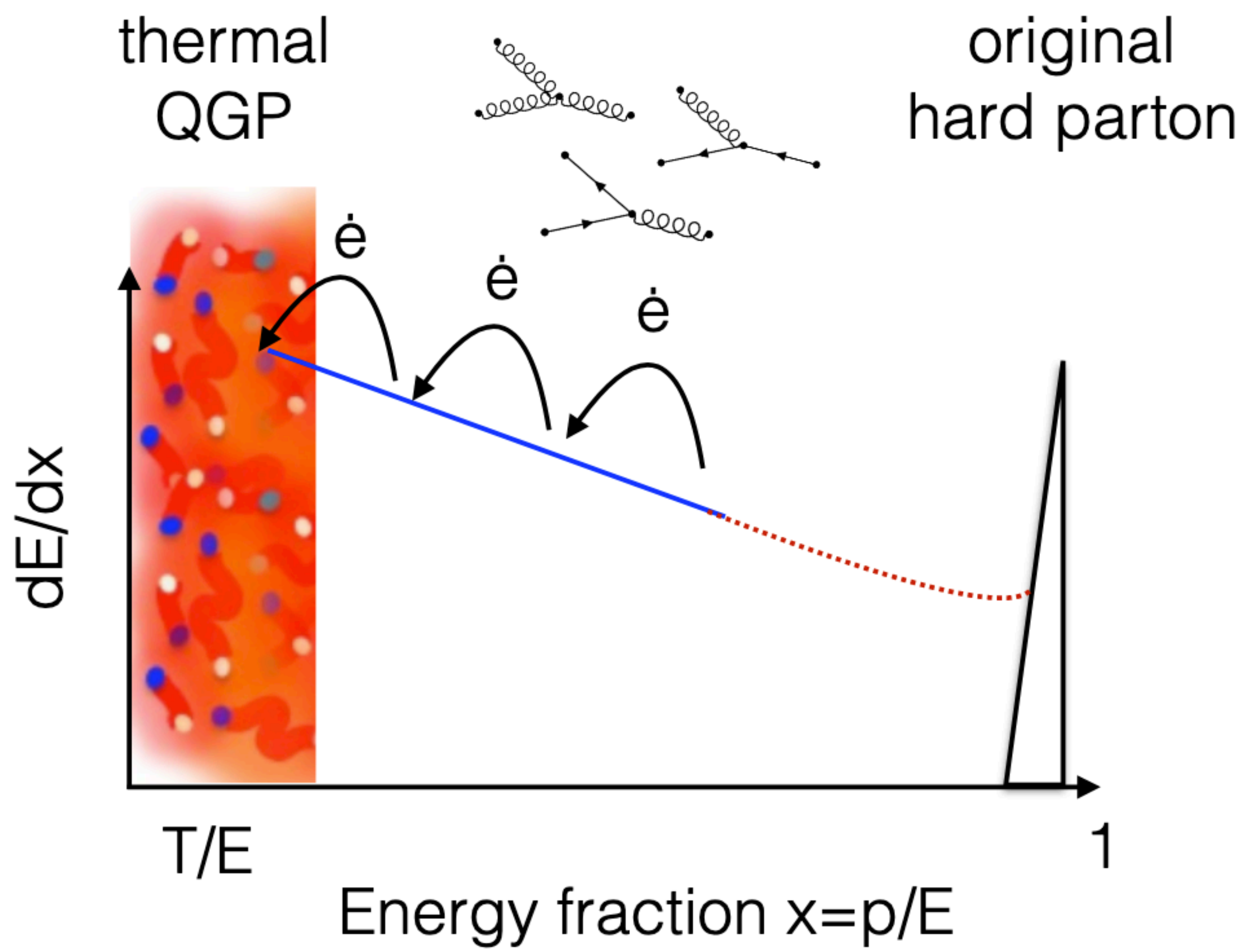
CRC-TR 211
Strong-interaction matter under extreme conditions

Medium energy loss

In-medium energy loss is dominated by an inverse energy cascade, driven by multiple successive splittings.

=>Requires a good grasp on the physics of in-medium splittings

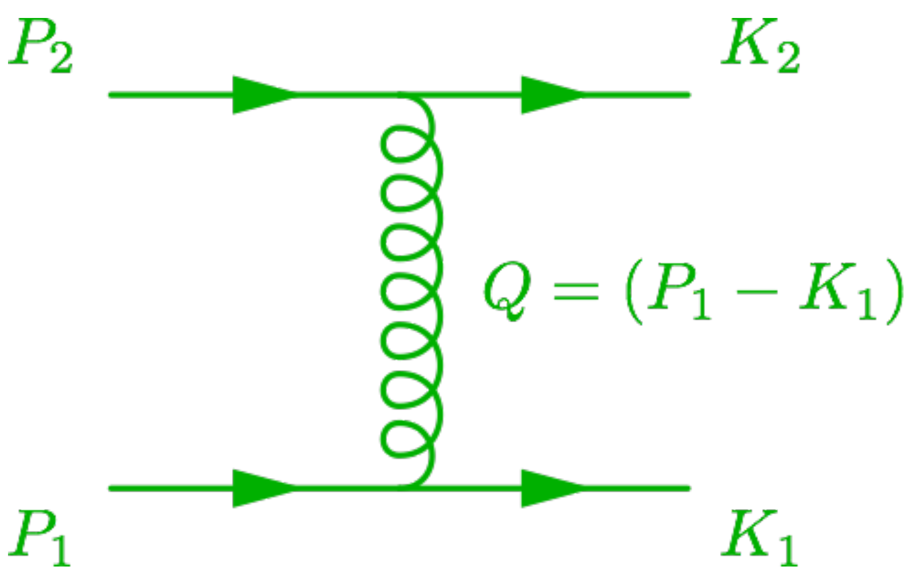


[see talk by S. Schlichting]

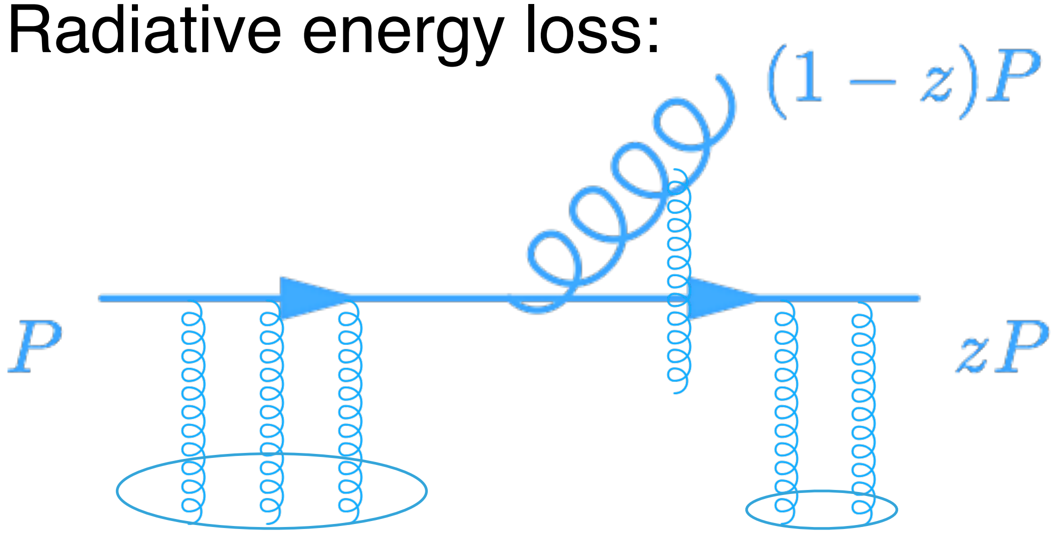
Jet studies

As the high energetic partons traverse the medium they lose energy due to:

- Elastic energy loss:



- Radiative energy loss:



Both require input from the medium transverse scattering rate :

$$\lim_{p \to \infty} \frac{d\Gamma(\boldsymbol{p}, \boldsymbol{p} + \boldsymbol{q}_\perp)}{d^2 q_\perp} = \frac{\mathcal{C}(q_\perp)}{(2\pi)^2}.$$

Transverse momentum broadening rate :

In the literature one employs pQCD broadening kernels:

- Static screened color centers -> $C(q) \propto \frac{1}{(q^2 + m_D^2)^2}$ [X. Wang and M. Gyulassy. *Phys.Rev.Lett.* 68 (1992) 1480-1483]
- Dynamics moving charges -> $C(q) \propto \frac{1}{q^2(q^2 + m_D^2)}$ [P. Aurenche, F. Gelis, and H. Zaraket. JHEP 05 (2002), p. 043.]
- Multiple soft scattering -> $C(b) \propto \frac{\hat{q}}{4} b^2$ [Baier-Dokshitzer-Mueller-Peigne-Schiff]

EQCD broadening kernel

Due to the infamous infrared problem of finite temperature QCD

=> perturbative calculations can receive large non-perturbative contribution even at small coupling.

$$C(\boldsymbol{b}_\perp) :\equiv \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} \left(1 - e^{-i\boldsymbol{q}_\perp \cdot b_\perp}\right) C(q_\perp) \, .$$

The collision kernel can be defined in terms of the behavior of certain light-like Wilson loops

$$C(\boldsymbol{b}_\perp) \equiv - \lim_{L\to\infty} \frac{1}{L} \ln \tilde{W}(L, \boldsymbol{b}_\perp) \, ,$$

[J. Casalderrey-Solana & D. Teaney, JHEP, vol. 04, p. 039, 2007]

=>For temperatures well above Tc these Wilson loops can be recast in the reduced effective theory of electrostatic QCD (EQCD)

[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

EQCD broadening kernel

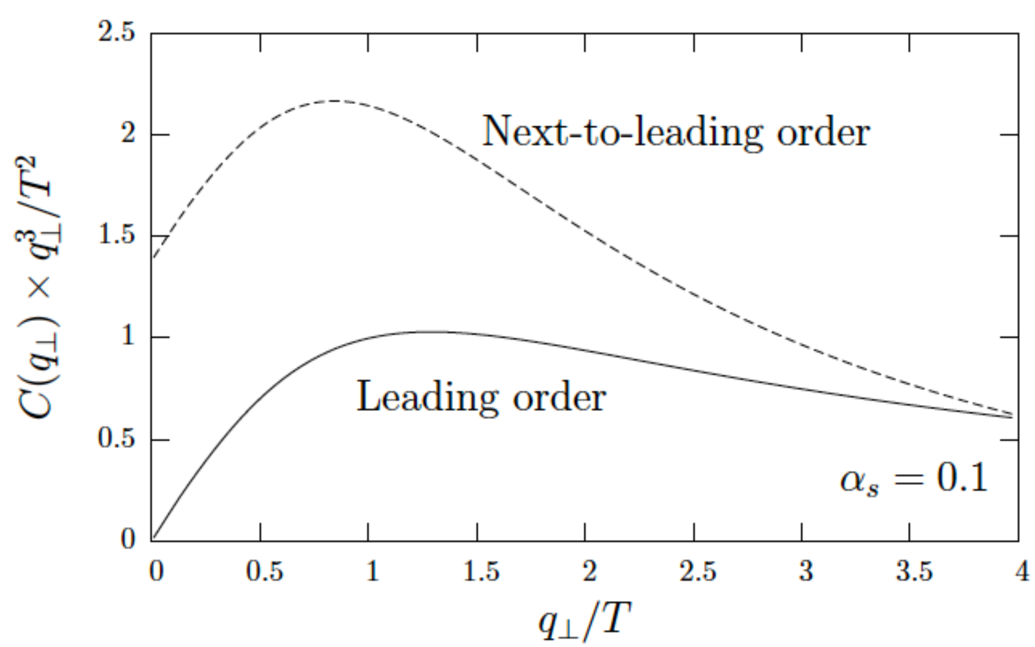
The kernel was computed in a perturbative expansion in the effective theory of EQCD :

- The LO EQCD kernel

$$C^{\rm LO}_{\rm QCD}(q_\perp) = \frac{g_s^4 T^3 C_{\rm R}}{q_\perp^2 (q_\perp^2 + m_D^2)} \int \frac{d^3p}{(2\pi)^3} \frac{p - p_z}{p} \left[2C_{\rm A} n_{\rm B}(p)\left(1 + n_{\rm B}(p')\right) + 4\, N_{\rm f} T_{\rm f}\, n_{\rm F}(p)\left(1 - n_{\rm F}(p')\right)\right] , = g_s^2 T C_{\rm R} \begin{cases} \frac{m_{\rm D}^2 - g_s^2 T^2 C_{\rm A} \frac{q_\perp}{16T}}{q_\perp^2 (q_\perp^2 + m_{\rm D}^2)} , & q_\perp \ll g_s T , \\ \frac{g_s^2 T}{q_\perp^4} \mathcal{N} , & q_\perp \gg g_s T , \end{cases}$$

[P. Arnold & W. Xiao Phys.Rev.D 78 (2008), 125008]

- NLO corrections :



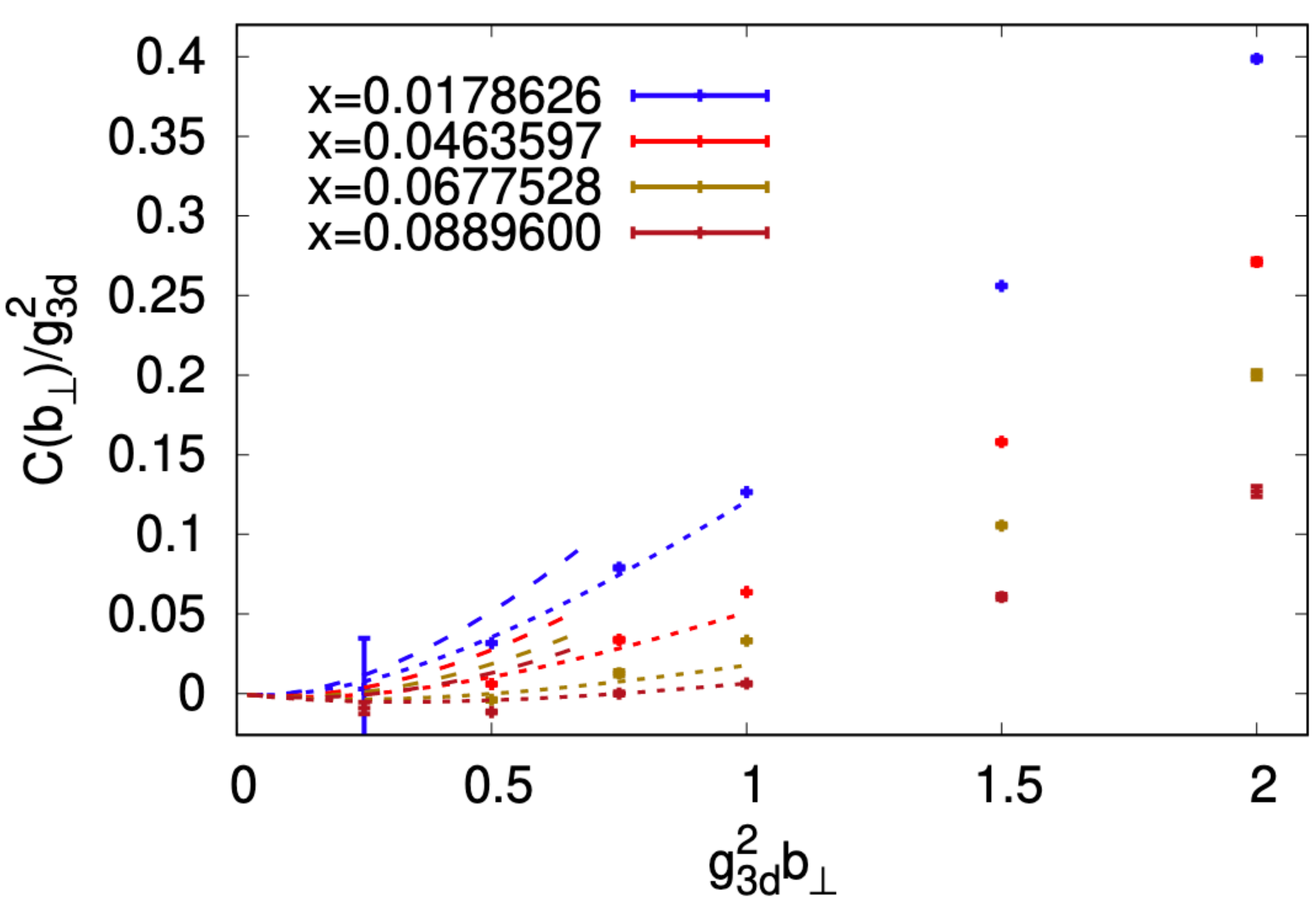
[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

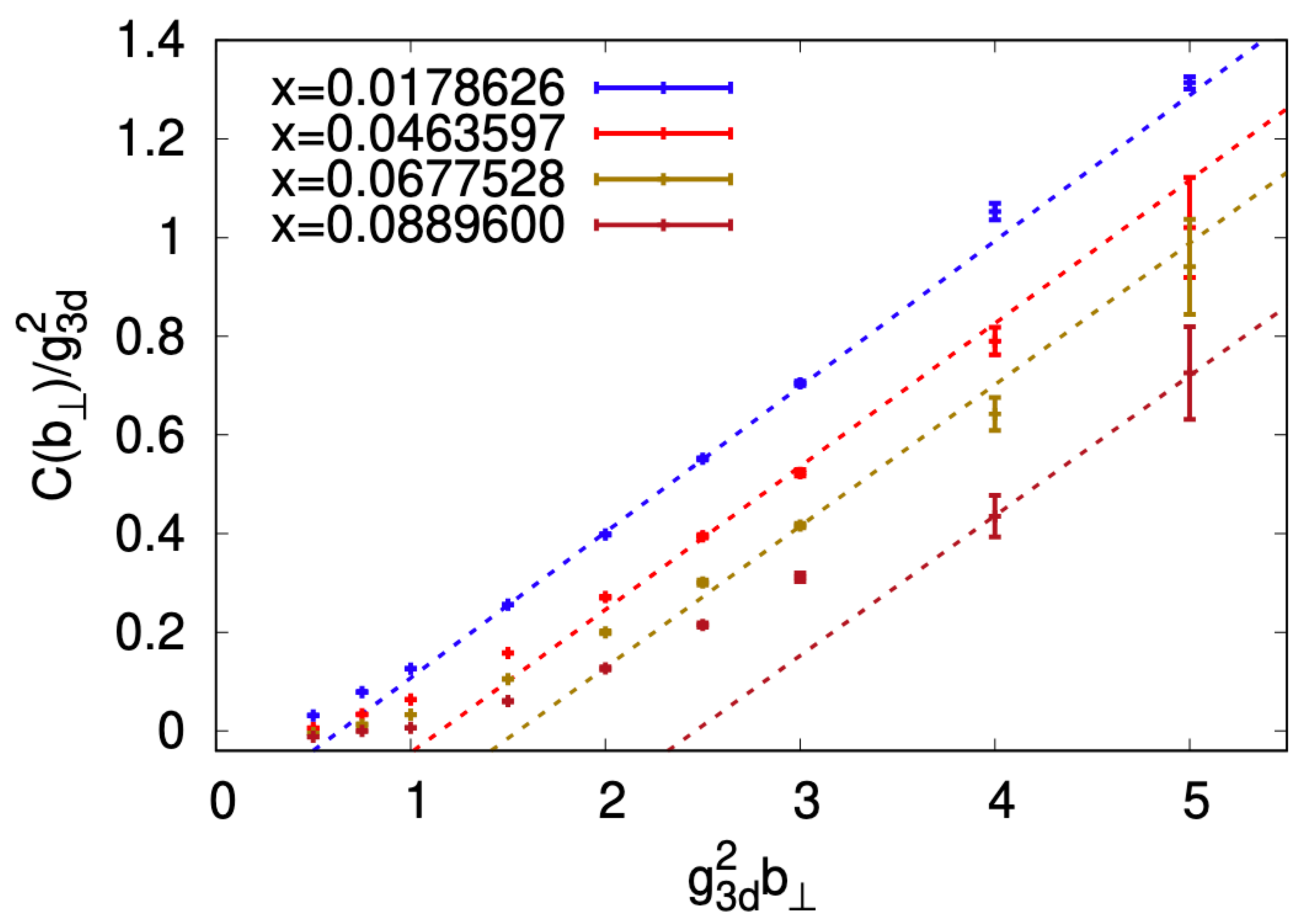
[see also talks by J. Ghiglieri and D. Teaney]

EQCD broadening kernel

Beyond the perturbative result, lattice extracted non-perturbative contribution were computed

[G.D. Moore & N. Schlusser Phys.Rev.D 101 (2020) 1, 014505]





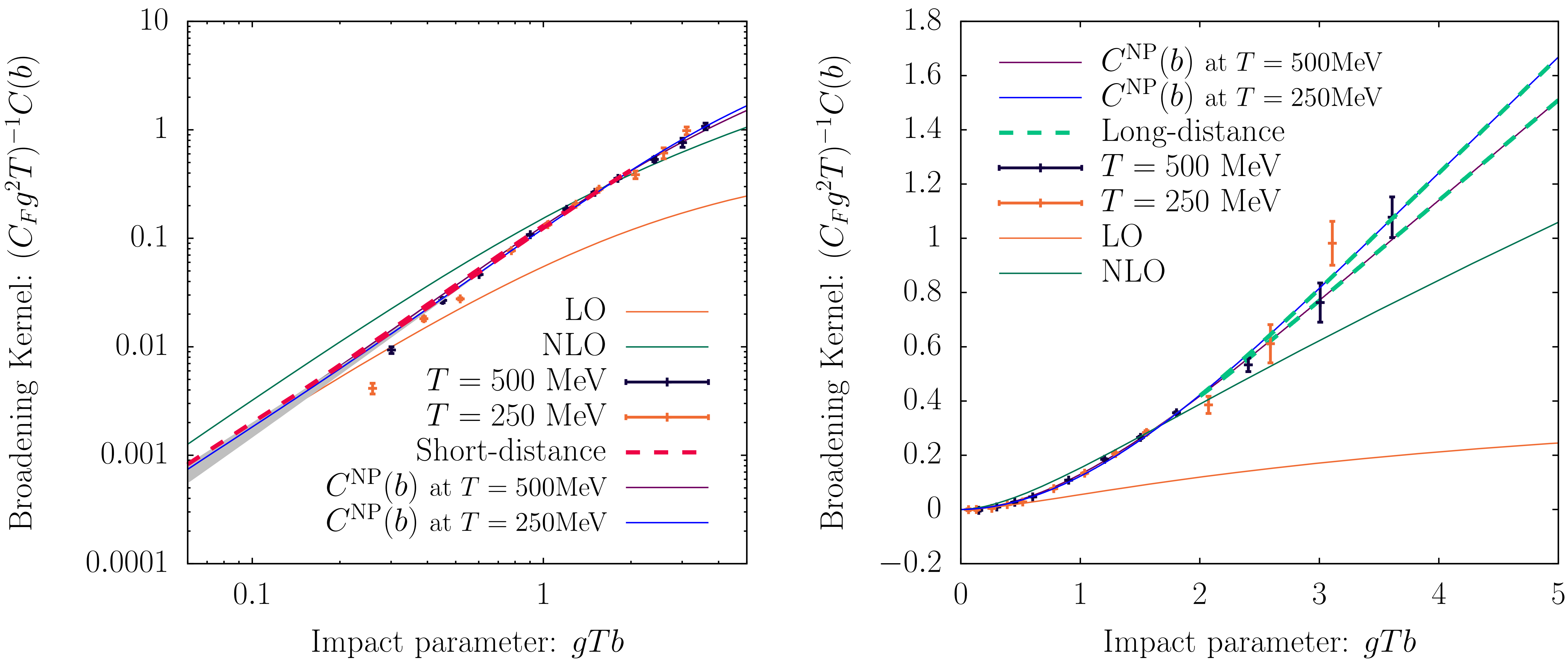
This result here is for the broadening kernel in EQCD which need to be matched to QCD

Non-perturbative broadening kernel

Since EQCD is a low-energy effective theory for QCD they should both agree in the IR regime but in the UV they can be different.

In order to ensure the right UV behavior while keeping the IR behavior from the lattice result we write the full kernel:

$$C_{\rm QCD}(b_\perp) \approx \left(C^{\rm pert}_{\rm QCD}(b_\perp) - C^{\rm pert}_{\rm EQCD}(b_\perp) \right) + C^{\rm latt}_{\rm EQCD}(b_\perp)\,.$$



[G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv:2105.01679]

Non-perturbative broadening kernel

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Long-distance behavior :

The kernel follows an area-law with sub-leading logarithm corrections

[M. Laine, Eur. Phys. J. C, vol. 72]

$$\frac{C_{\rm QCD}}{g_{\rm 3d}^2}(b_\perp) \xrightarrow{b_\perp \gg 1/g_{\rm 3d}^2} \boxed{A + \frac{\sigma_{\rm EQCD}}{g_{\rm 3d}^4} g_{\rm 3d}^2 b_\perp} + \frac{g_{\rm s}^4 C_{\rm R}}{\pi}\left[\frac{y}{4}\left(\frac{1}{6} - \frac{1}{\pi^2}\right) + \frac{C_{\rm A}}{8\pi^2 g_{\rm s}^2}\right]\log(g_{\rm 3d}^2 b_\perp)\,,$$

Short-distance behavior :

The kernel follows the same behavior as the LO one, where we determine $\hat{q}_0$ from the data :

$$\frac{C_{\rm QCD}}{g_{\rm 3d}^2}(b_\perp) \xrightarrow{b_\perp \ll 1/m_{\rm D}} -\frac{C_{\rm R}}{8\pi}\frac{\zeta(3)}{\zeta(2)}\left(-\frac{1}{2g_{\rm s}^2} + \frac{3y}{2}\right)(g_{\rm 3d}^2 b_\perp)^2 \log(g_{\rm 3d}^2 b_\perp) + \frac{1}{4}\frac{\hat{q}_0}{g_{\rm 3d}^6}(g_{\rm 3d}^2 b_\perp)^2\,,$$

In-medium splitting rates

Using the broadening kernel at hand, one can compute in-medium splitting rates

- In the AMY approach:

[Arnold-Moore-Yaffe]

$$\frac{d\Gamma_{ij}}{dz}(P,z) = \frac{\alpha_s P_{ij}(z)}{[2Pz(1-z)]^2} \int \frac{\mathrm{d}^2\mathbf{p}_\perp}{(2\pi)^2} \, \mathrm{Re}\left[2\mathbf{p}_\perp \cdot \mathbf{g}_{(z,P)}(\mathbf{p}_\perp)\right]$$

where $g_{(z,P)}(p_\perp)$ is solution to the integral equation :

$$\begin{aligned} 2\mathbf{p}_\perp = i\delta E(z,P,\mathbf{p}_\perp)\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) + \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} \, \bar{C}(q_\perp) \\ \times \left\{ C_1 \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - \mathbf{q}_\perp)\right] \right. \\ + C_z \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - z\mathbf{q}_\perp)\right] \\ \left. + C_{1-z} \left[\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - (1-z)\mathbf{q}_\perp)\right] \right\} . \end{aligned}$$

In-medium splitting rates

BH regime :

$$Pz(1-z) \ll \omega_{\rm BH} \sim T$$

Formation time is small and interference between scatterings can be neglected.

One solves the rate equations in opacity expansion in the number of elastic scatterings with medium.

Amounts to calculating a numerical integral.

[Gyulassy-Levai-Vitev]

LPM regime :

$$Pz(1-z) \gg \omega_{\rm BH} \sim T$$

Typical number of rescatterings within the formation time of bremsstrahlung can be large.

Interference of many soft scatterings need to be considered.

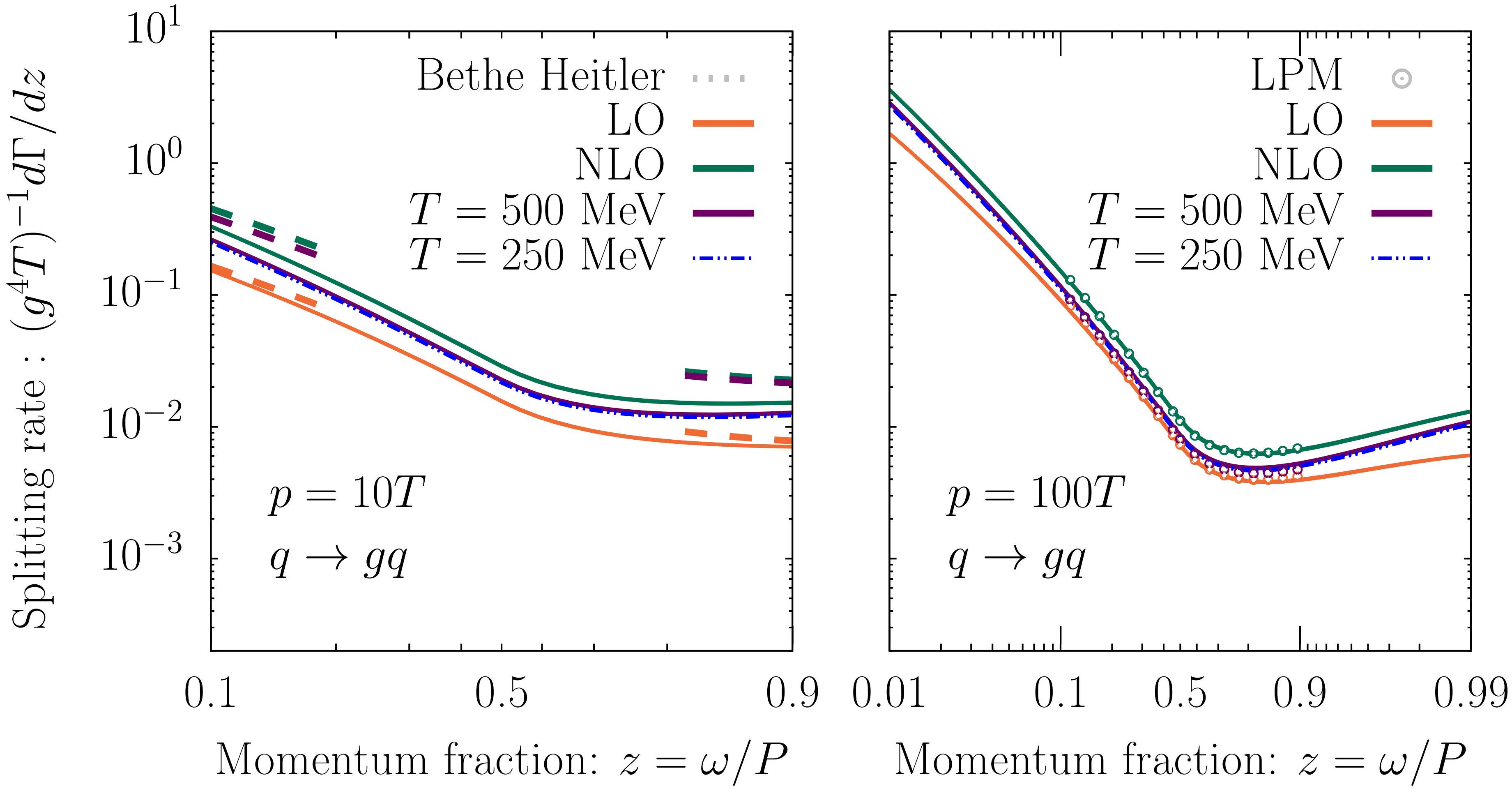
The broadening kernel follows a $b_\perp^2$ behavior at small distances

$$C(b_\perp) = -\frac{g_s^2 T^2}{16\pi} \mathcal{N} b_\perp^2 \log(\xi m_D^2 b_\perp^2/4)$$

The rate equations become a Harmonic oscillator problem and solved analytically

[P. Arnold and C. Dogan. *Phys. Rev. D* 78 (2008), p. 065008]

Results for non-perturbative splitting rates



- The two temperatures (250 MeV and 500 MeV) do not display a remarkable difference
- Recover LPM suppression at large momentum.
- In the large energy region the rate is closer to the LO one as they both follow the same behavior in broadening kernel at short distances

Finite medium

To compute the rate in finite medium it is best to work in momentum space

The rate follows the limits :

- $q \gg 1$:

$$C^{\mathrm{UV}}(q_\perp) = \frac{C_{\mathrm{R}}}{8\pi}\frac{\zeta(3)}{\zeta(2)}\left(-\frac{1}{2g_s^2} + \frac{3y}{2}\right)\frac{8\pi}{q_\perp^4}\,.$$

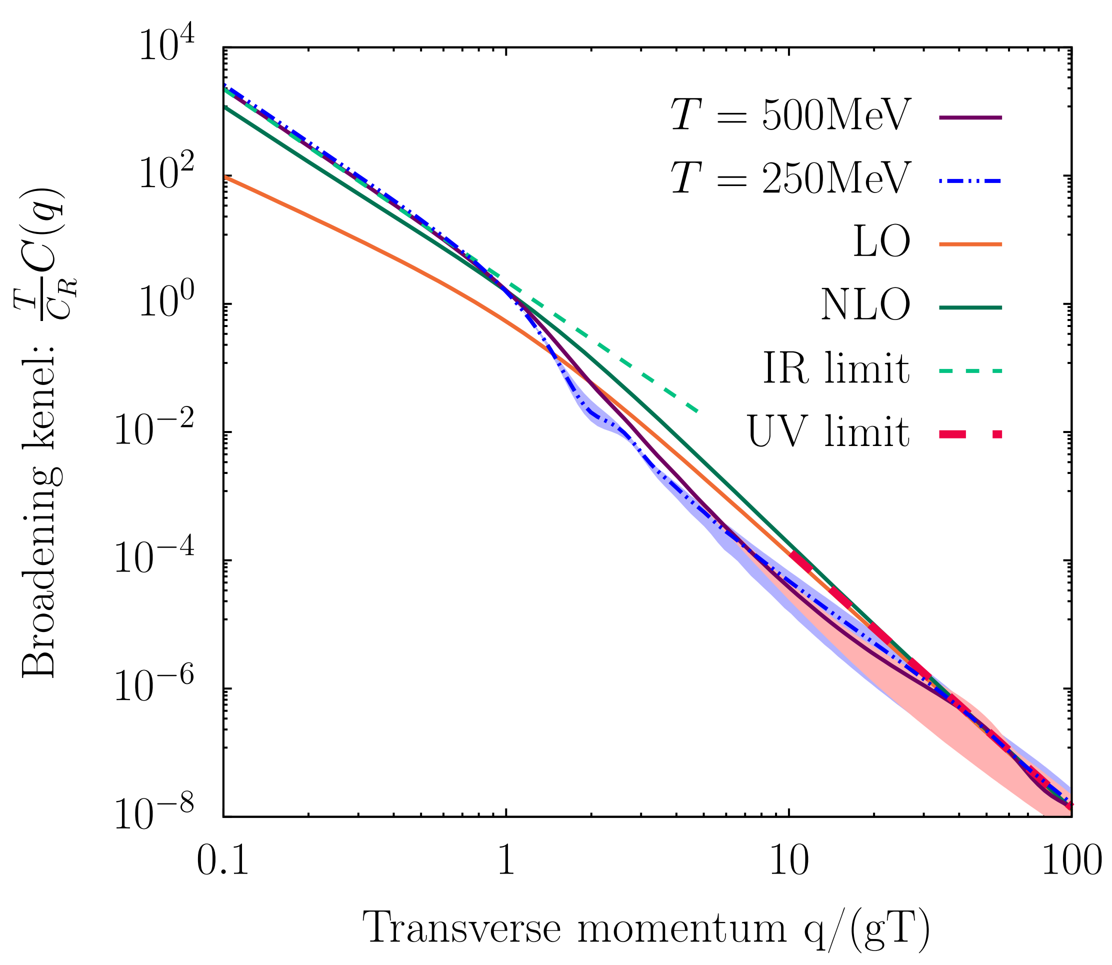
Similar to the LO rate

- $q \ll 1$:

$$C^{\mathrm{IR}}(q_\perp) = \frac{2\pi}{q_\perp^3}\frac{\sigma_{\mathrm{EQCD}}}{g_{3d}^2} + \frac{g^4 C_{\mathrm{R}}}{\pi}\left[\frac{y}{4}\left(\frac{1}{6} - \frac{1}{\pi^2}\right) + \frac{C_{\mathrm{A}}}{8\pi^2 g_s^2}\right]\frac{2\pi}{q_\perp^2}\,. \tag{6}$$

stems from the area law with string tension

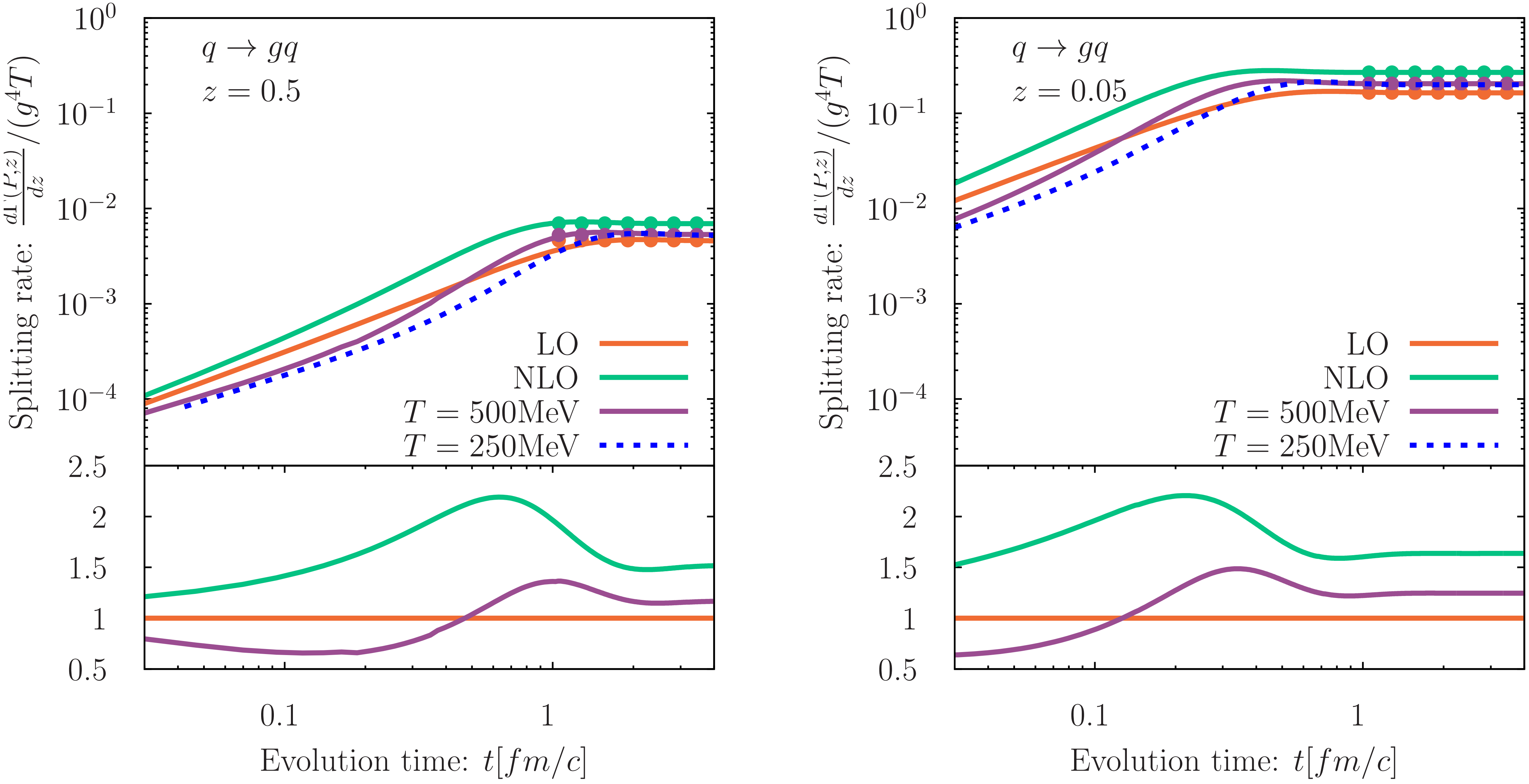
Transverse Momentum Broadening:



Preliminary results in finite medium

- We compute the rate in finite-medium following the approach of S. Caron-Huot and C. Gale [S. Caron-Huot, C. Gale Phys.Rev.C 82 (2010), 064902]

Medium-induced splitting rates: $P = 300T$



Resummed opacity expansion

- We compare the finite medium with an improved opacity expansion where after we cut low momentum interactions, we exponentiate higher order of the expansion

$$\left.\frac{d\Gamma^a_{bc}}{dz}\right|_{N=X}(P,z,\tilde{t}) = \frac{g^4 T P^a_{bc}(z)}{\pi}\mathrm{Re}\int_0^{\tilde{t}} d\Delta\tilde{t}\int_{\tilde{\boldsymbol{p}}} e^{-(i\delta\tilde{E}(\tilde{p})+\Lambda\Sigma_3(\tilde{p}^2))\Delta\tilde{t}}\tilde{\psi}_I^{(1)}(\tilde{p})\ ,$$

[C. Andres et Al. JHEP 03 (2021), 102]

where the first order wave function is the collision integral of the initial condition

$$\psi^{(1)}(\boldsymbol{p}) = \int_{\boldsymbol{q}} n(t)C(\boldsymbol{q})\left\{C_1\left[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{p^2-\boldsymbol{p}\cdot\boldsymbol{q}}{\epsilon(\boldsymbol{p}-\boldsymbol{q})}\right]\right.$$
$$\left. + C_z\left[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{p^2+z\boldsymbol{p}\cdot\boldsymbol{q}}{\epsilon(\boldsymbol{p}+z\boldsymbol{q})}\right] + C_{1-z}\left[\frac{p^2}{\epsilon(\boldsymbol{p})} - \frac{(p^2+(1-z)\boldsymbol{p}\cdot\boldsymbol{q})}{\epsilon((\boldsymbol{p}+(1-z)\boldsymbol{q}))}\right]\right\}$$

and the subsequent interactions are encoded in the exponential of

$$\Sigma(M^2) \equiv \int_{\vec{k}^2 > M^2} n(t)C(\vec{k})(C_1 + C_z + C_{1-z})$$

Improved opacity expansion

- Improved opacity expansion around the Harmonic Oscillator

[Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk]

$$C(b_\perp) = \frac{g_s^4 T^3}{16\pi}\mathcal{N} b_\perp^2 \ln\left(\frac{4Q^2}{\xi m_D^2}\right) + \frac{g_s^4 T^3}{16\pi}\mathcal{N} b_\perp^2 \ln\left(\frac{1}{Q^2 b_\perp^2}\right) = C^{HO}(b_\perp) + C^{\text{pert}}(b_\perp) \,,$$

$$\frac{dI^{NLO}}{dz}(P,z,t) = \frac{dI^{HO}}{dz}(P,z,t) + \frac{dI^{(1)}}{dz}(P,z,t) \,.$$

- By self-consistently solving for the scale

$$Q^2(P,z) = \sqrt{Pz(1-z)\hat{q}_3(Q^2)} \,,$$

$$\hat{q}_{\text{eff}}(Q^2) = \frac{g_s^4 T^3}{4\pi}\mathcal{N}\left[C_1 + C_z z^2 + C_{1-z}(1-z)^2\right] \ln\left(\frac{4Q^2}{\xi m_D^2}\right) \,,$$

- One can find correction to the HO

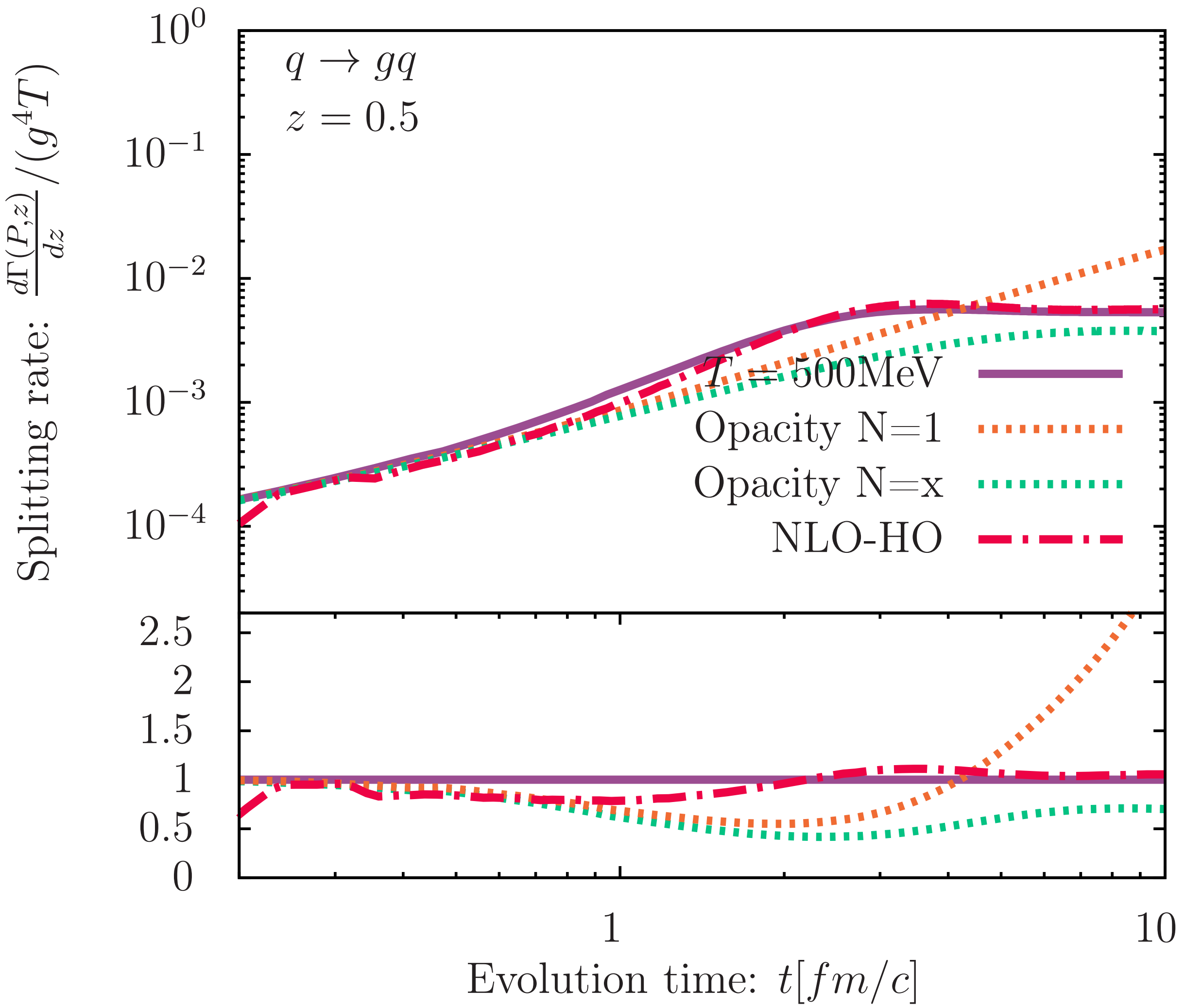
$$\frac{dI^{HO}}{dz}(P,z,t) = \frac{g^2}{4\pi^2} \ln|\cos\Omega t| \,,$$

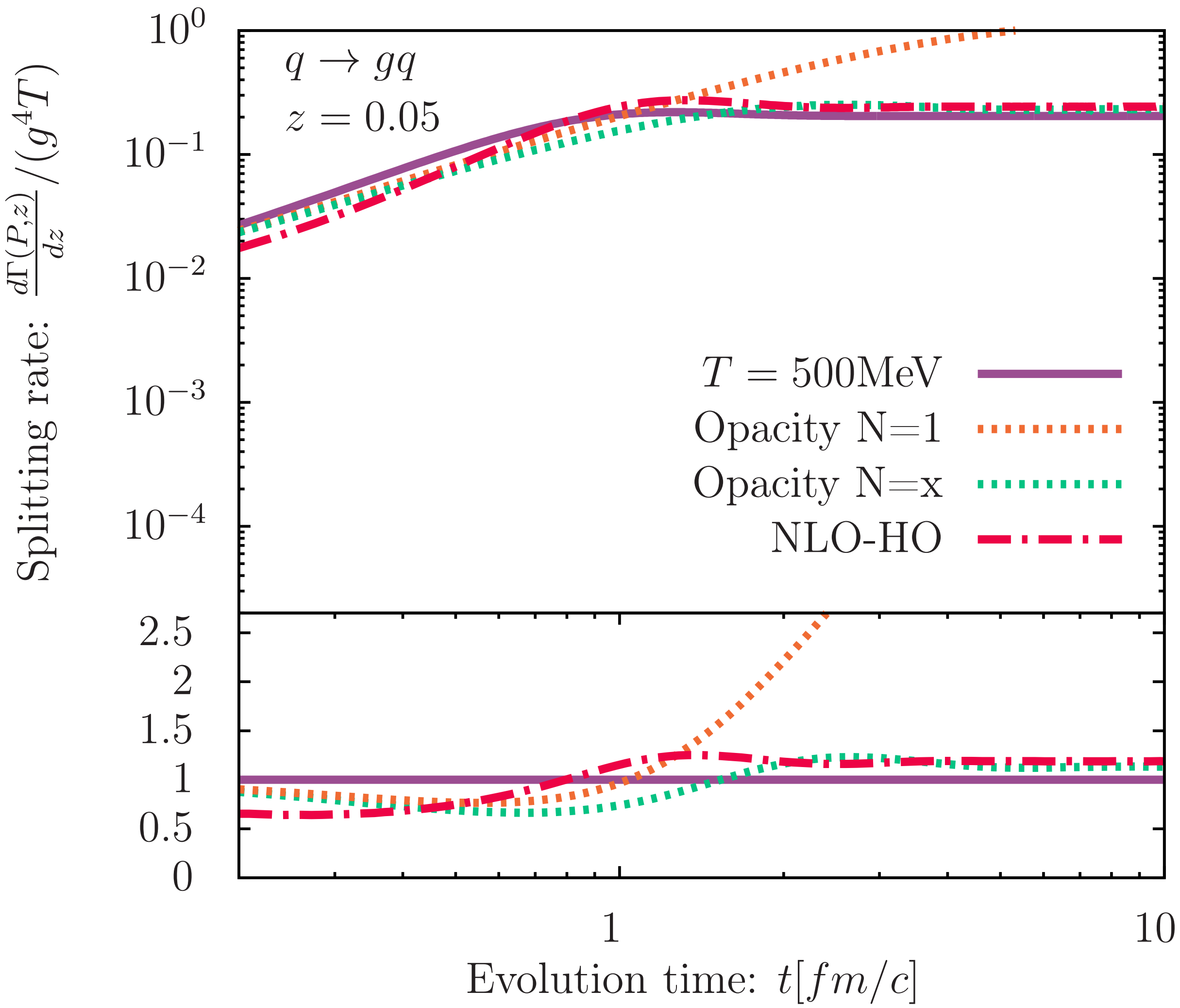
$$\frac{dI^{(1)}}{dz}(P,z,t) = \frac{g^2}{4\pi^2}\text{Re}\int_0^t ds \int_0^\infty \frac{2du}{u}\left[C_1 C^{(1)}(u) + C_z C^{(1)}(zu) + C_{1-z} C^{(1)}((1-z)u)\right] e^{k^2(s)u^2} \,,$$

[See talks by C. Andreas and K. Tywoniuk]

Preliminary results in finite medium

Medium-induced splitting rates: $P = 300T$





[G.D. Moore & N. Schlusser Phys.Rev.D 102 (2020) 9, 094512]

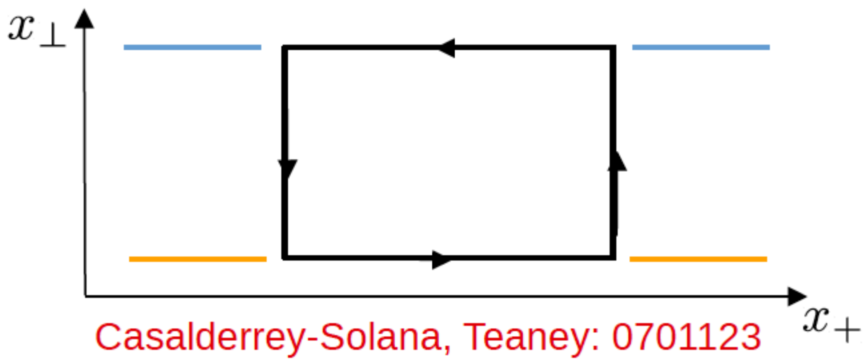
- Collision kernel

$$C(q_\perp) = \frac{\mathrm{d}\Gamma}{\mathrm{d}^2 q_\perp \, \mathrm{d}L}$$

- Asymptotic mass

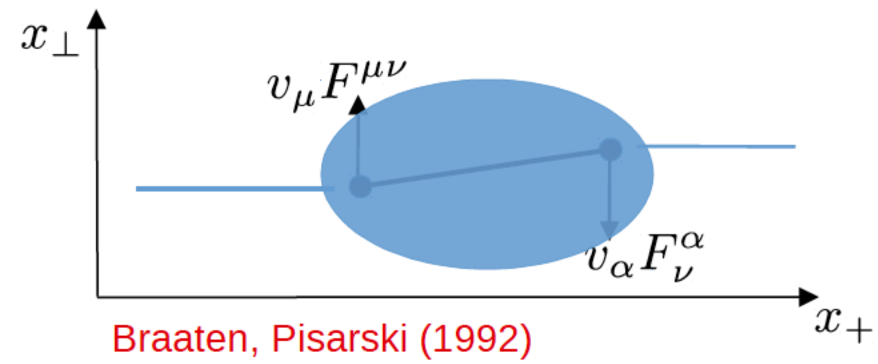
$$m_\infty^2 = C_\mathrm{R}\,(Z_\mathrm{g} + Z_\mathrm{f})$$

- Wilson loop



Casalderrey-Solana, Teaney: 0701123

- Force-force-correlator



Braaten, Pisarski (1992)

Nonperturbative gluon-zero-mode contributions:

→ calculate in lattice EQCD

Caron-Huot: 0811.1603

Slide courtesy of N. Schlusser

Conclusion & Outlook

- Using non-perturbative contribution, we observe a sizable difference to the perturbative broadening kernel as well as to the in-medium splitting rates (~25%)
- Although the limiting behaviors are recovered in the limiting cases, a robust full calculation of the rate is important.

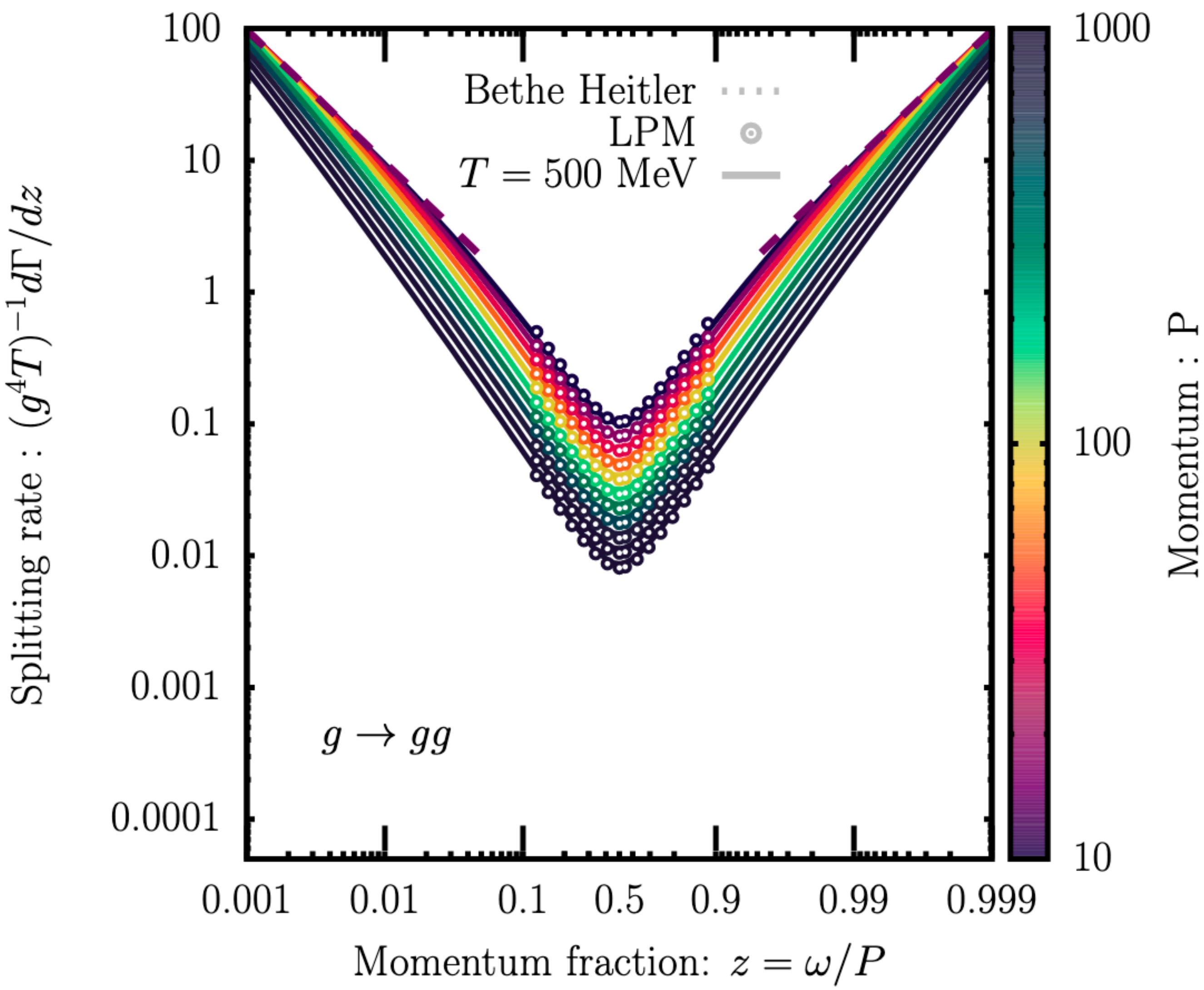
[G.D. Moore & N. Schlusser Phys.Rev.D 102 (2020) 9, 094512]

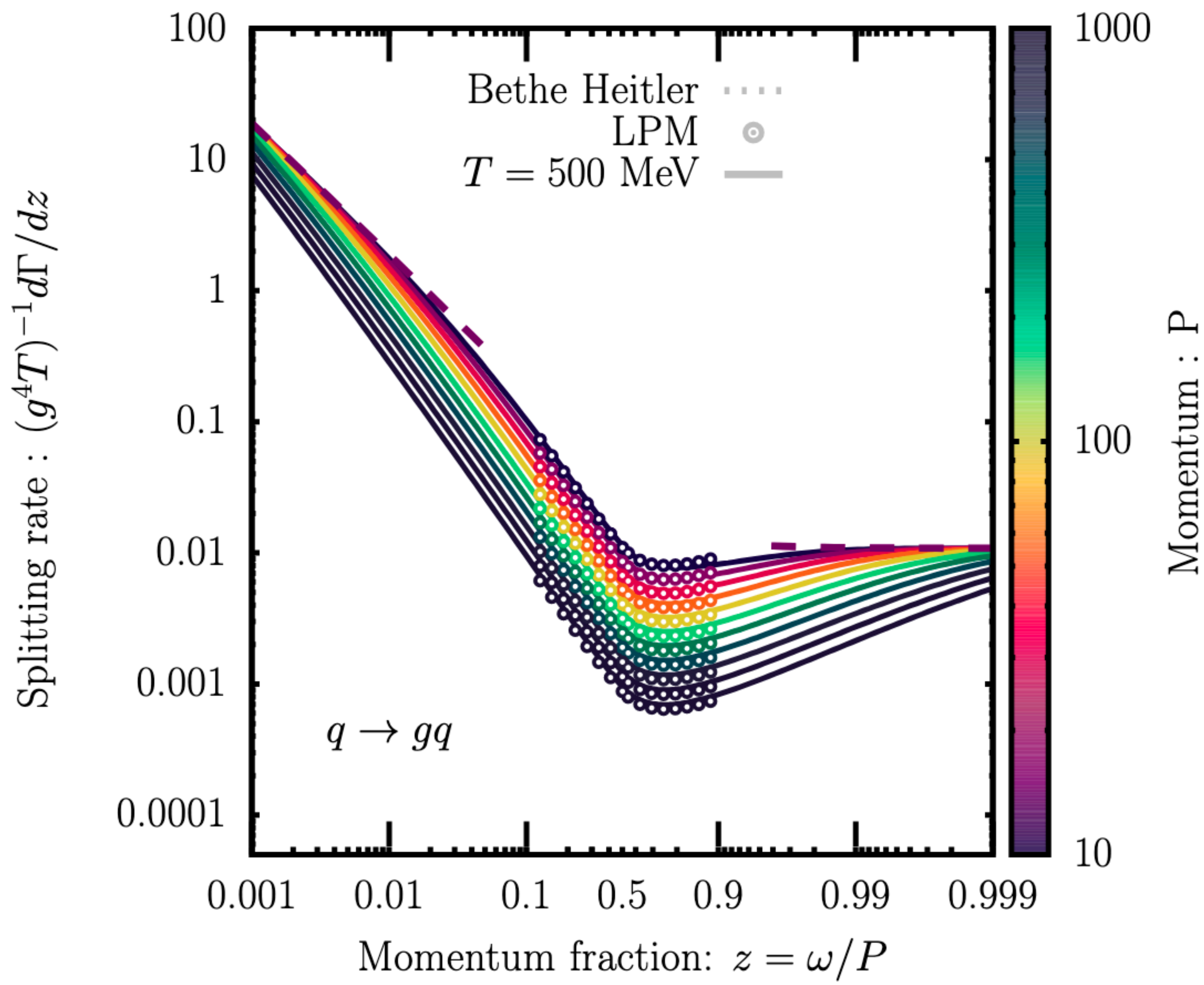
- There have been effort to extract asymptotic masses using the same procedure which still needs to be matched to QCD.
- Would be interesting to include the non-perturbative results to jet studies (elastic and radiative interactions) in kinetic studies or MC

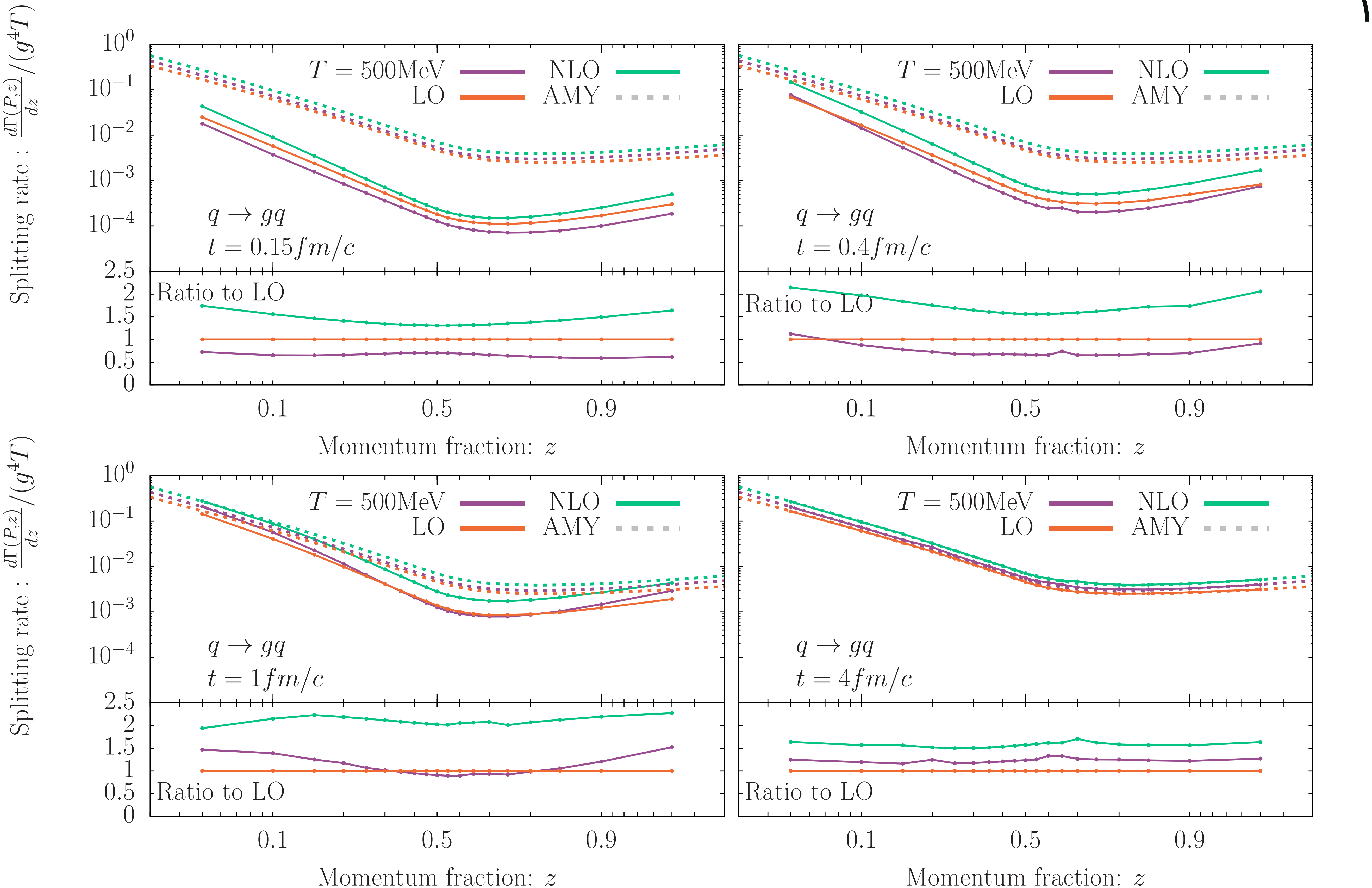
Thank you!

Backup

Results for non-perturbative splitting rates







Splitting rate : $\frac{d\Gamma(P,z)}{dz}/(g^4T)$
$T = 500$MeV
NLO
LO
AMY
Ratio to LO
$q \to gq$
$t = 0.15fm/c$
$t = 0.4fm/c$
$t = 1fm/c$
$t = 4fm/c$
Momentum fraction: z