Finite size effects on jet thermalization

Ismail Soudi

University of Jyväskylä
Helsinki Institute of Physics

Based on:

- WIP
- IS arXiv:2409.04806
- Y. Mehtar-Tani, S. Schlichting, and IS JHEP 05 (2023) 091
- S. Schlichting, IS JHEP 07 (2021), 077

UNIVERSITY OF JYVÄSKYLÄ

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Yocto/LHC
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Centre of Excellence in Quark Matter

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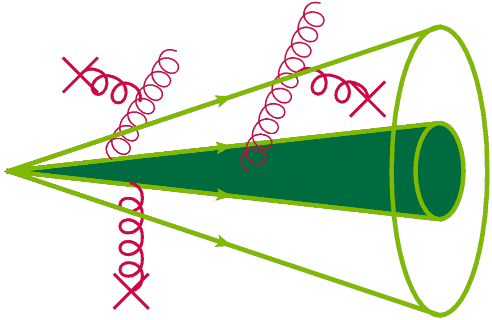
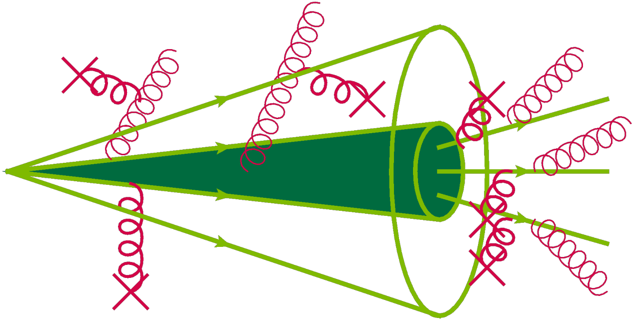
Introduction I



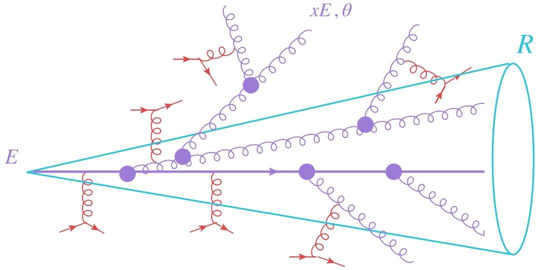
- Hard parton evolution in heavy ion collisions

⊙ Introduction I



- Hard parton evolution in heavy ion collisions
- Resolved by the medium ⇒ Energy loss / thermalization

⊙ In-Medium Shower



- Main focus: Hard parton traversing a QCD plasma
- Understand: Energy cascade, out-of-cone energy loss, medium response and full thermalization of the shower ⇒ Important for low energy jets

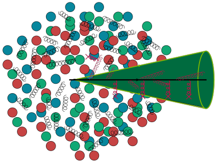
⊙ Outline

- ⊙ Introduction
- ⊙ **Effective Kinetic Description**
- ⊙ Longitudinal Energy Loss
- ⊙ Thermalization To Large Angles
- ⊙ Beyond AMY

⊙ Effective Kinetic Description

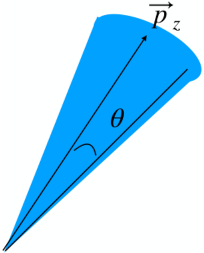
- **Leading Order Effective Kinetic Theory:**

$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = C_i[\{f_i\}] \ , \quad (1)$$



- **Hard partons $\Rightarrow$ Linearized Fluctuation on top of Static Equilibrium**

$$f(p,t) = n_{\mathrm{eq}}(p,t) + \delta f_{\mathrm{jet}}(p,t) \ , \quad (2)$$



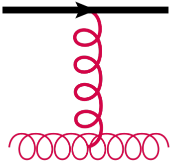
- **Energy distribution**

$$D(x,\theta,t) \equiv x \frac{dN_a}{dx d\cos\theta} \sim \frac{\nu_a(N_f)}{E_j} \delta f_{\mathrm{jet}}(p,\theta,t) \ . \quad (3)$$

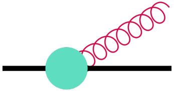
- **Exact conservation of energy, momentum and valence charge $\rightarrow$ Study evolution from $\sim E$ to $\sim T$ including full thermalization**

AMY EKT

- Elastic Scatterings using HTL screened $2 \leftrightarrow 2$



- Collinear Radiation effective $1 \leftrightarrow 2$



$$C[\{f_i\}] = \qquad C^{2\leftrightarrow 2}[\{f_i\}] \qquad + \qquad C^{1\leftrightarrow 2}[\{f_i\}]\,. \tag{4}$$

- Analogous to QGP thermalization in pre-Hydro phase
- Using AMY rates for in-medium radiation: $t_{\text{form}} \sim \sqrt{\frac{z(1-z)P}{\hat{q}}} \ll L$

Elastic Scattering

Elastic scattering collision integral with HTL screened matrix elements

$$C_a^{2\leftrightarrow 2}[\{f_i\}] = \frac{1}{2|p_1|\nu_a} \sum_{bcd} \int d\Omega^{2\leftrightarrow 2} |\mathcal{M}^{ab}_{cd}(p_1, p_2, p_3, p_4)|^2 \delta\mathcal{F}(p, p_1, p_2, p_3, p_4) \tag{5}$$

Full detailed balance terms $\Rightarrow$ Medium response

$$\begin{aligned} \delta F(p_1, p_2, p_3, p_4) = \quad & \delta f_a(p_1)[\pm_a n_c(p_3) n_d(p_4) - n_b(p_2)(1 \pm n_c(p_3) \pm n_d(p_4))] \\ & + \delta f_b(p_2)[\pm_b n_c(p_3) n_d(p_4) - n_a(p_1)(1 \pm n_c(p_3) \pm n_d(p_4))] \\ & - \delta f_c(p_3)[\pm_c n_a(p_1) n_b(p_2) - n_d(p_4)(1 \pm n_a(p_1) \pm n_b(p_2))] \\ & - \delta f_d(p_4)[\pm_d n_a(p_1) n_b(p_2) - n_c(p_3)(1 \pm n_a(p_1) \pm n_b(p_2))] \,. \end{aligned} \tag{6}$$

Collinear Radiation

Collinear radiation (including BH & LPM effects)

$$
\begin{aligned}
C_g^{g\leftrightarrow gg}[\{D_i\}] = \int_0^1 dz \frac{d\Gamma^g_{gg}\left(\left(\frac{xE}{z}\right), z\right)}{dz} \Bigg[& D_g\left(\frac{x}{z}\right)\left(1 + n_B(xE) + n_B\left(\frac{\bar{z}xE}{z}\right)\right) \\
& + \frac{D_g(x)}{z^3}\left(n_B\left(\frac{xE}{z}\right) - n_B\left(\frac{\bar{z}xE}{z}\right)\right) + \frac{D_g\left(\frac{\bar{z}xE}{z}\right)}{\bar{z}^3}\left(n_B\left(\frac{xE}{z}\right) - n_B(xE)\right)\Bigg] \\
& - \frac{1}{2}\int_0^1 dz \frac{d\Gamma^g_{gg}(xE, z)}{dz}\Bigg[D_g(x)(1 + n_B(zxE) + n_B(\bar{z}xE)) \\
& + \frac{D_g(zx)}{z^3}(n_B(xE) - n_B(\bar{z}xE)) + \frac{D_g(\bar{z}x)}{\bar{z}^3}(n_B(xE) - n_B(zxE))\Bigg] ,
\end{aligned}
$$

- **AMY Radiation Rate**

$$
\frac{d\Gamma^a_{bc}}{dz}(P, z, \infty) = \frac{g^2 P^a_{bc}(z)}{4\pi P^2 z^2 (1-z)^2} \mathrm{Re} \int_0^\infty dt_1 \int_{\boldsymbol{p},\boldsymbol{q}} \frac{i\boldsymbol{q}.\boldsymbol{p}}{\delta E(\boldsymbol{q})} \Gamma_3(t) \circ G(\infty, \boldsymbol{q}; t_1, \boldsymbol{p}) \, . \qquad (7)
$$

- **Merging/Splitting ⇒ Thermalization of soft sector**

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- Introduction
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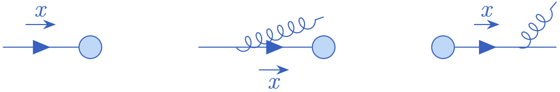
- Beyond AMY

Longitudinal Energy Loss

- Evolution can be divided into three regimes:
  1. Initial energy loss: mediated by single gluon radiation
  2. Energy cascade: successive emissions lead to an energy cascade
  3. Equilibration: Late time decay to soft sector

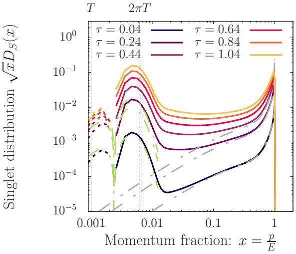
⊙ Single Emission

- **Initial distribution determined from direct emissions**



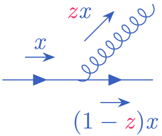
$$D_q(x,\tau) \simeq \delta(1-x) \quad + \left[x \frac{d\Gamma^q_{gq}(x)}{dz} \quad - \int_0^1 dz \;\; z \frac{d\Gamma^q_{gq}(x,z)}{dz} \delta(1-x) \right] \tau \qquad (8)$$

- **Short lived stage ⇒ Subsequent emissions lead to energy cascade**



⊙ Kolmogorov-Zakharov Turbulence

- Take simple HO kernel



$$\frac{d\Gamma(xE,z)}{dz} \simeq \frac{1}{x^\alpha}\mathcal{K}(z)\,, \quad D(x,s) = x^\beta G\,. \tag{9}$$

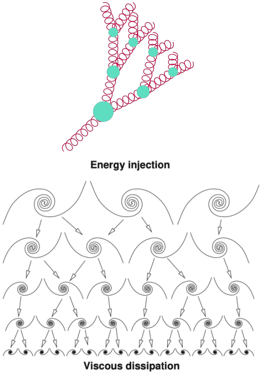
- The energy flux:

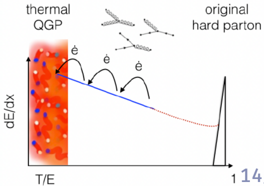
$$\frac{dE}{d\tau} \overset{\mu\to 0}{=} -\int_{\mu/E}^{1} dz \;\; z\mathcal{K}(z)\int_{\mu/E}^{\mu/zE} dx \;\; x^{\beta-\alpha} G\,. \tag{10}$$

- If $\beta-\alpha=-1$

$$\int_{\mu/E}^{\mu/zE} dx \;\; x^{-1} = \Big[\ln(x)\Big]_{\mu/E}^{\mu/zE} = \ln(z)\,, \tag{11}$$

scale drops out ⇒ constant Flux [a]



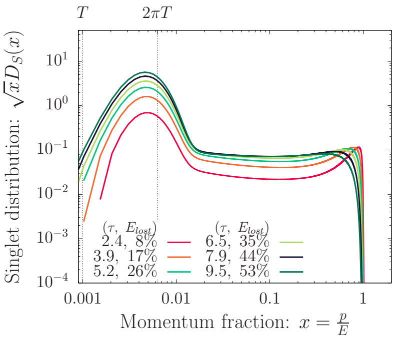


[a] Blaizot, Iancu, Mehtar-Tani PRL(2013), Blaizot, Mehtar-Tani AoP(2015), Mehtar-Tani, Schlichting JHEP(2018), Schlichting, IS JHEP(2021)

Kolmogorov-Zakharov Turbulence

- **Stationary turbulent solution in intermediate energy range** $T/E \ll x \ll 1$:

$$D_g(x) = \frac{G}{\sqrt{x}}\,, \qquad D_S(x) = \frac{S}{\sqrt{x}}\,. \qquad (9)$$



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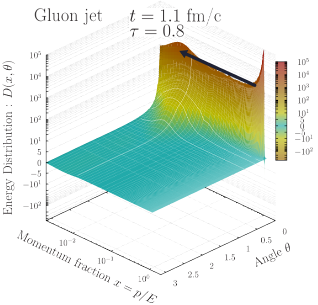
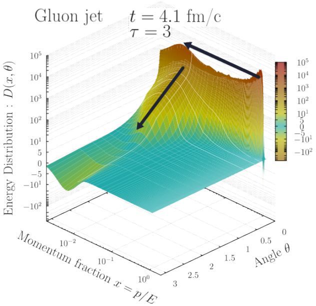
⊙ Thermalization To Large Angles

- **Energy cascade to soft sector**

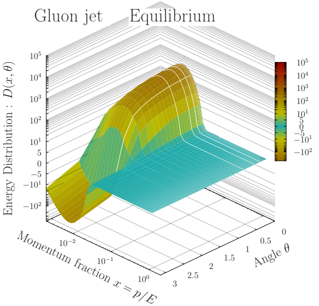


[1]

- **Broadening of soft partons to large angle**



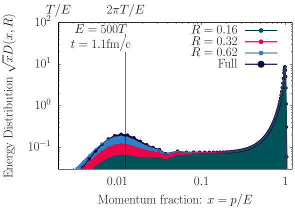
- **Thermalization of soft sector**



[1] $E/T = 500, \quad g = 2.$

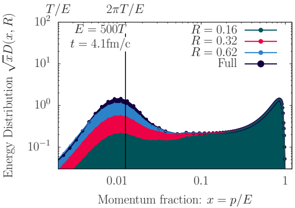
Thermalization To Large Angles

- **Energy cascade to soft sector**

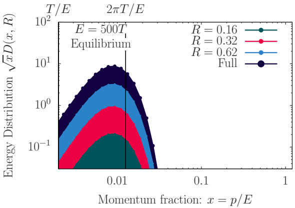


[1]

- **Broadening of soft partons to large angle**



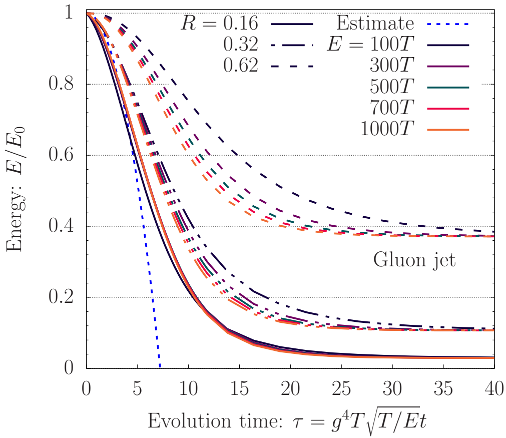
- **Thermalization of soft sector**



[1] $E/T = 500, \quad g = 2.$

⊙ Sensitivity To Initial Parton's Energy

- Characteristic time of the turbulent cascade $t_{th} = \frac{1}{\alpha_s}\sqrt{\frac{E}{\hat{q}}}$
- Scaling between different initial parton's energy for small cone-sizes
- Broken for large cone-sizes



Modeling Jet Quenching

- In the presence of QGP, the jet spectrum factorizes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \int_0^\infty \mathrm{d}\epsilon \;\; P(\epsilon, R) \frac{\mathrm{d}\sigma_{\text{vac}}}{\mathrm{d}p_T^{in}}(p_T^{in} \equiv p_T + \epsilon)\,, \tag{10}$$

- The energy loss probability $P(\epsilon, R)$ is obtained using the BDMPS rate

$$P(\epsilon, R) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \omega_i \frac{\mathrm{d}I}{\omega_i} \mathrm{d}\omega_i\right] \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i\right) \exp\left[-\int_0^\infty \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\right] \tag{11}$$

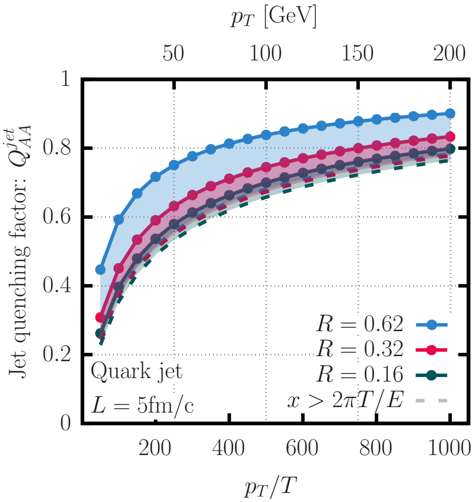
- Using a Milne transform $\Rightarrow$ Exponential
- Taking vacuum spectrum $\frac{\mathrm{d}\sigma_{\text{vac}}}{\mathrm{d}p_T^{in}}(p_T^{in}) \propto (p_T^{in})^{-\alpha}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = Q(p_T, R) \frac{\mathrm{d}\sigma_{\text{vac}}}{\mathrm{d}p_T}\,, \quad \Rightarrow \quad Q(p_T, R) \approx \exp\left[-\int \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\left(1 - \mathrm{e}^{-n\omega/p_T}\right)\right] \tag{12}$$

⊙ Modeling Jet Quenching

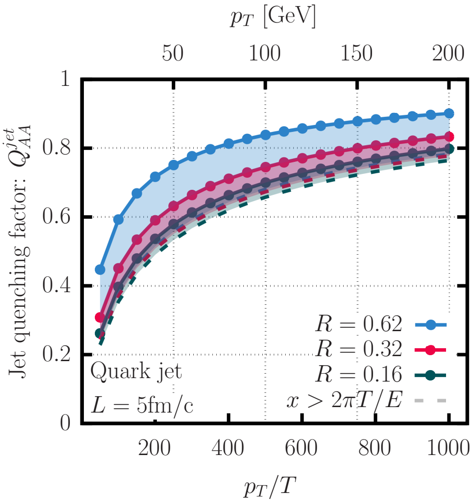
- **The first emission is modeled using BDMPS finite medium rate $\frac{d\Gamma}{d\omega}(P,\omega,t)$ at time t**
- **Medium energy loss computed by modeling the energy remaining inside the cone $E(\omega,R,L-t)$ after a time $(L-t)$ in the medium**

$$Q(p_T) = \exp\left[\int_0^L \mathrm{d}t \int \mathrm{d}\omega \right.$$
$$\left. \times \frac{\mathrm{d}\Gamma}{\mathrm{d}\omega}\left(1-\mathrm{e}^{-n\frac{\omega}{p_T}\left[1-\frac{E\left(\omega|R,\tau=\frac{L-t}{t_{\mathrm{th}}}\right)}{\omega}\right]}\right)\right]. \quad (13)$$



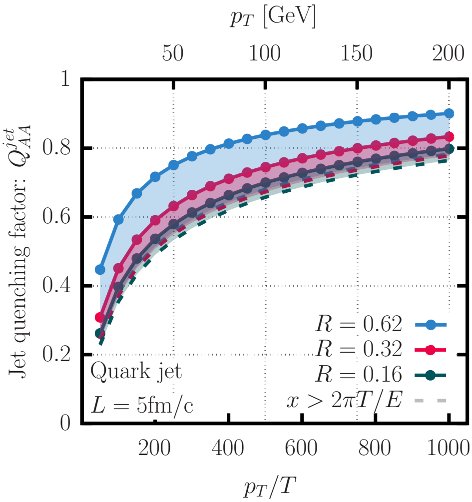
Modeling Jet Quenching

- Jets with energies $p_T \leq 25$GeV lose significant energy in length $L = 5fm/c$
  $\Rightarrow$ large suppression at low p_T, milder for $p_T \leq 200 - 400$GeV
- Negligible contribution of soft fragments to narrow cone
- Large cone size (≥ 0.3) $\Rightarrow$ Recover energy from the soft sector
  - ▷ Medium response



⊙ Modeling Jet Quenching

- ⚠ Over-estimate energy loss since we neglect finite size effects, Work in progress
- ⚠ Requires more refined studies of near-equilibrium physics and jet recoil onto the medium



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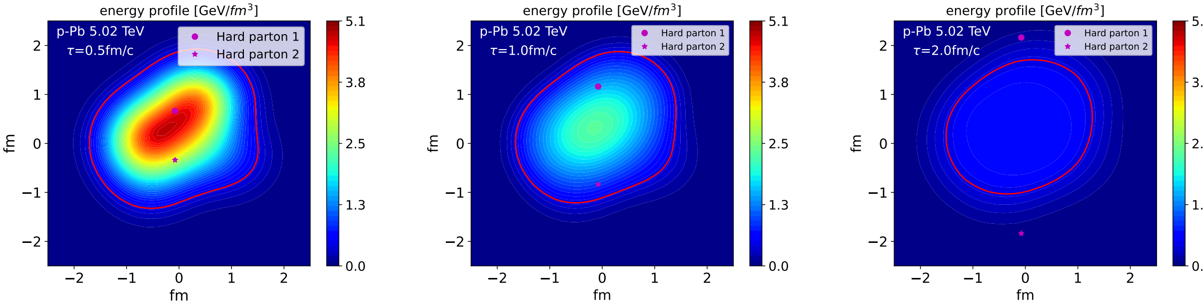
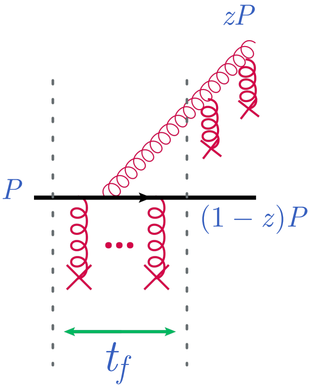
⊙ Energy loss in small systems?



- System small/short lived ⇒ Expect minimal energy loss

[2] I. Soudi et al. (2024). In: arXiv: 2407.17443.

⊙ Collinear Radiation



- Multiple scatterings ⇒ induced radiation
- Emission controlled by the formation time

$$t_{\rm form} \sim \frac{z(1-z)xE}{k_T^2} \quad , \quad k_T^2 \sim \hat{q}\, t_{\rm form} \Rightarrow t_{\rm form} \sim \sqrt{\frac{z(1-z)xE}{\hat{q}}} \, , \tag{13}$$

- $t_{\rm form} \ll \lambda_{\rm mfp}$: Medium cannot resolve the quanta until it is formed
- $t_{\rm form} \gg \lambda_{\rm mfp}$: Multiple scatterings act coherently

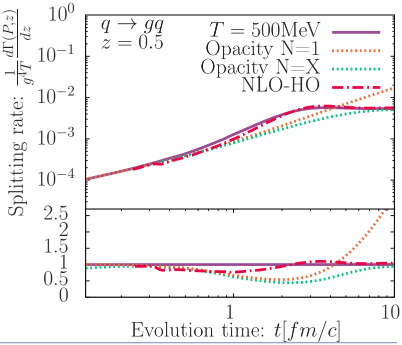
- Quantum interference ⇒ suppression of high energy radiation ⇒ LPM effect

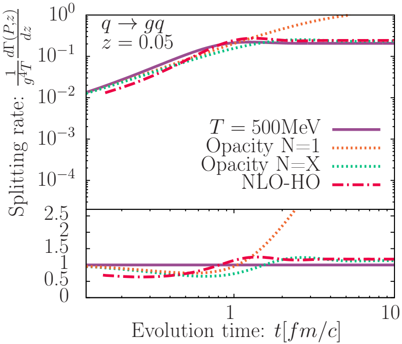
[2](Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov, Wiedemann, Arnold, Moore, Yaffe..)

⊙ Finite Size Effects: Rates

$$\frac{d\Gamma^a_{bc}}{dz}(P,z,t) = \frac{g^2 P^a_{bc}(z)}{4\pi P^2 z^2(1-z)^2}\mathrm{Re}\int_0^t dt_1 \int_{\boldsymbol{p},\boldsymbol{q}} \frac{i\boldsymbol{q}.\boldsymbol{p}}{\delta E(\boldsymbol{q})}\Gamma_3(t)\circ G(\infty,\boldsymbol{q};t_1,\boldsymbol{p})\,. \quad (14)$$

- **Long formation time:** $z \simeq 0.5$
- **Short formation time:** $z(1-z) \simeq 1$





[2] Caron-Huot, Gale PRC(2010), Schlichting, IS PRD(2022)

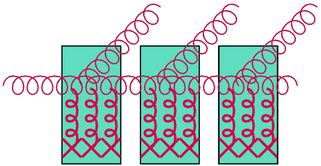
Finite Size Effects: Evolution I

- Including finite size effects in the evolution
- Global formation time:

$$\partial_t D(x) = \int_0^1 dz \left[\frac{d\Gamma^a_{bc}}{dz} \left(\frac{x}{z}, z, \boldsymbol{t} \right) zD\left(\frac{x}{z}\right) - \frac{d\Gamma^a_{bc}}{dz}(x, z, \boldsymbol{t}) zD(x) \right]$$

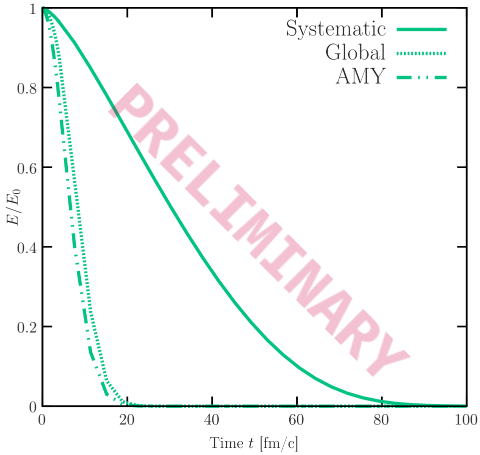
- Systematic formation time:

$$\partial_t D(x) = \int_0^t ds \int_0^1 dz \left[\frac{d\Gamma^a_{bc}}{dz} \left(\frac{x}{z}, z, \boldsymbol{t-s} \right) zD\left(\frac{x}{z}\right) - \frac{d\Gamma^a_{bc}}{dz}(x, z, \boldsymbol{t-s}) zD(x) \right]$$



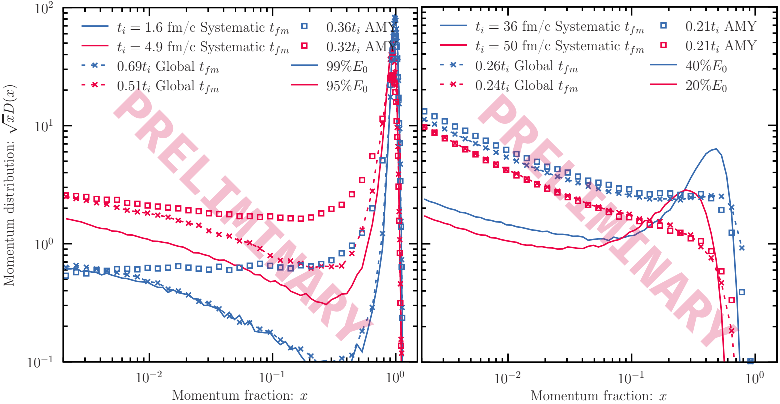
[2] Barata, Domínguez, Salgado, Vila JHEP(2021)

⊙ Finite Size Effects: Results I



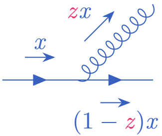
- Delay of energy loss due to formation time

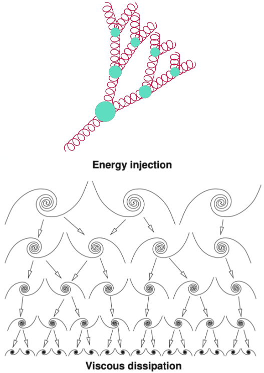
⊙ Finite Size Effects: Results II



- Markedly different evolution for Systematic treatment of formation time

⊙ Kolmogorov-Zakharov Turbulence





- Take simple HO kernel

$$\frac{d\Gamma(xE, z \quad)}{dz} \simeq \frac{1}{x^{\alpha}} \mathcal{K}(z \quad) \,, \quad D(x, s) = x^{\beta} G \,. \tag{15}$$

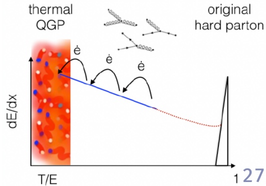
- The energy flux:

$$\frac{dE}{d\tau} \overset{\mu\to 0}{=} - \int_{\mu/E}^{1} dz \; z \quad \mathcal{K}(z, \qquad) \int_{\mu/E}^{\mu/zE} dx \; x^{\beta-\alpha} \qquad G \,. \tag{16}$$

- If $\beta - \alpha \quad = -1$

$$\int_{\mu/E}^{\mu/zE} dx \; x^{-1} = \Big[\ln(x)\Big]_{\mu/E}^{\mu/zE} = \ln(z) \,, \tag{17}$$

scale drops out $\Rightarrow$ constant Flux



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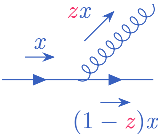


$$\frac{d\Gamma(xE,z,t)}{dz} \simeq \frac{1}{x^{\alpha(t-s)}}\mathcal{K}(z,t)\,, \quad D(x,s)=x^{\beta}G\,. \tag{15}$$

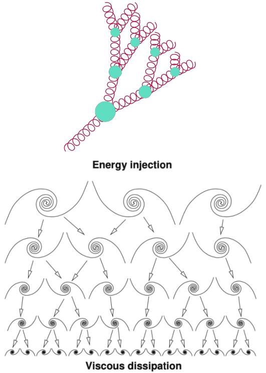
- The energy flux:

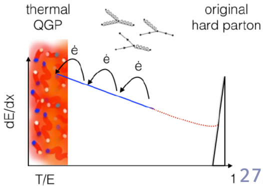
$$\frac{dE}{d\tau} \overset{\mu\to 0}{=} -\int_0^t ds \int_{\mu/E}^{1} dz \;\; z\,\partial_t \mathcal{K}(z,t-s)\int_{\mu/E}^{\mu/zE} dx \;\; x^{\beta-\alpha(t-s)} G\,. \tag{16}$$

- If $\beta - \alpha(t-s) = -1$

$$\int_{\mu/E}^{\mu/zE} dx \;\; x^{-1} = \Big[\ln(x)\Big]_{\mu/E}^{\mu/zE} = \ln(z)\,, \tag{17}$$

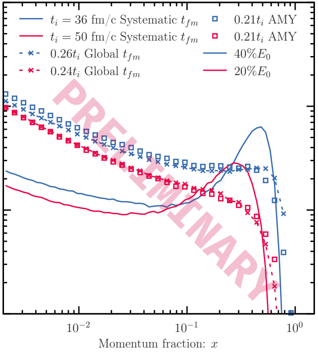
scale drops out $\Rightarrow$ constant Flux



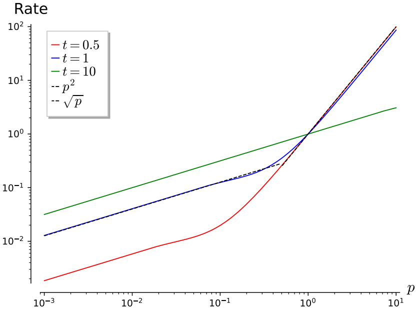


⊙ Kolmogorov-Zakharov Turbulence

- **Stationary turbulence**

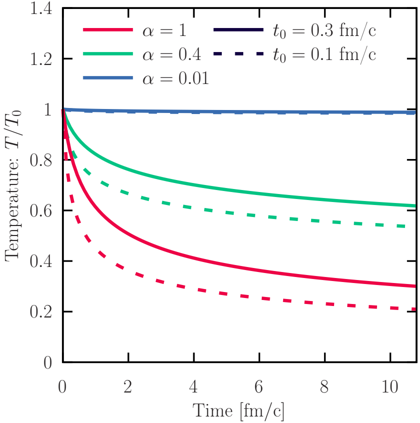


- **x-dependence for HO as a function of t**



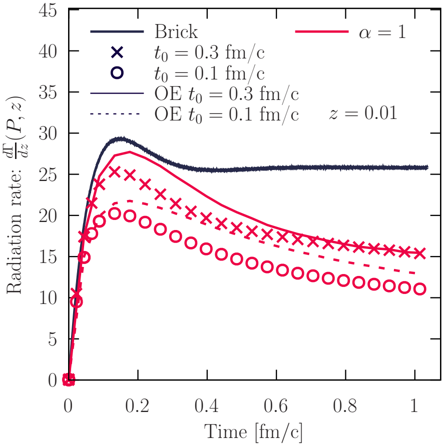
⊙ Bjorken Expansion: Medium-Induced Radiation I

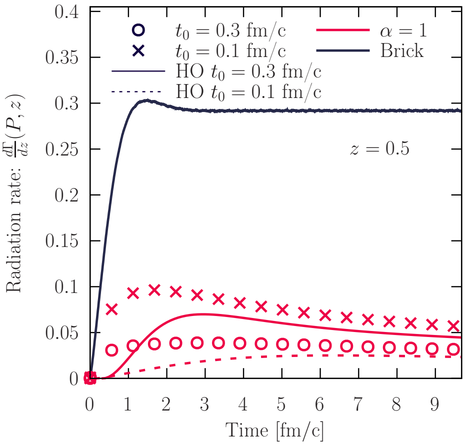
- **Bjorken expansion:** $T(t) = T_0 \left(\frac{t_0}{t+t_0}\right)^{1/3}$



⊙ Bjorken Expansion: Medium-Induced Radiation II

- **Bjorken expansion:** $T(t) = T_0 \left(\frac{t_0}{t+t_0}\right)^{1/3}$





Conclusion

- Energy loss is governed by an inverse energy cascade
  $\Rightarrow$ driven by successive collinear radiation
- Energy deposited collinearly at the soft scales rapidly broadens to large angles
- Formation time lead to dramatic effect on medium-cascade $\Rightarrow$ Delay of energy loss, survival of high energy partons

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Thank you for your attention \o/
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Cone-Size Dependence

—●— : $E(R,\tau) = \int_0^\infty dx \int_{\cos R}^1 d\cos\theta \;\; D(x,\cos\theta,\tau)$ (15)

- Small cone-sizes: Soft sector does not play major role
- Large cone-sizes: Energy loss display different behavior ⇒ Dominated by thermalization

-·-·- : $E_{2\pi}(R,\tau) = \int_{2\pi T/E}^\infty dx \int_{\cos R}^1 d\cos\theta \;\; D(x,\cos\theta,\tau)$ (16)