

Nonperturbative phenomena in jet modification

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Based on: G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv: 2105.01679
S. Schlichting, I.S. work in progress

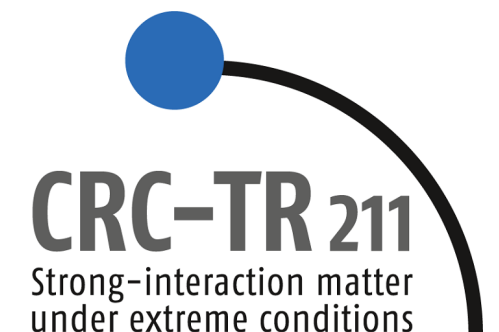
A Virtual Tribute to Quark Confinement and the Hadron Spectrum 2021,
August 5th, 2021

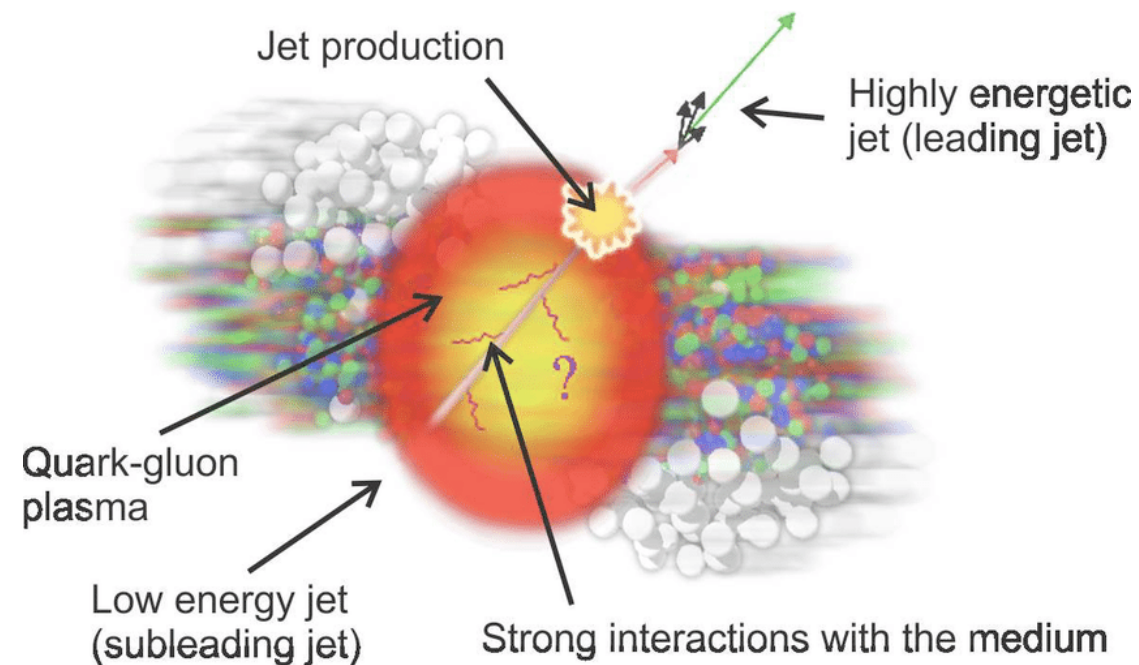


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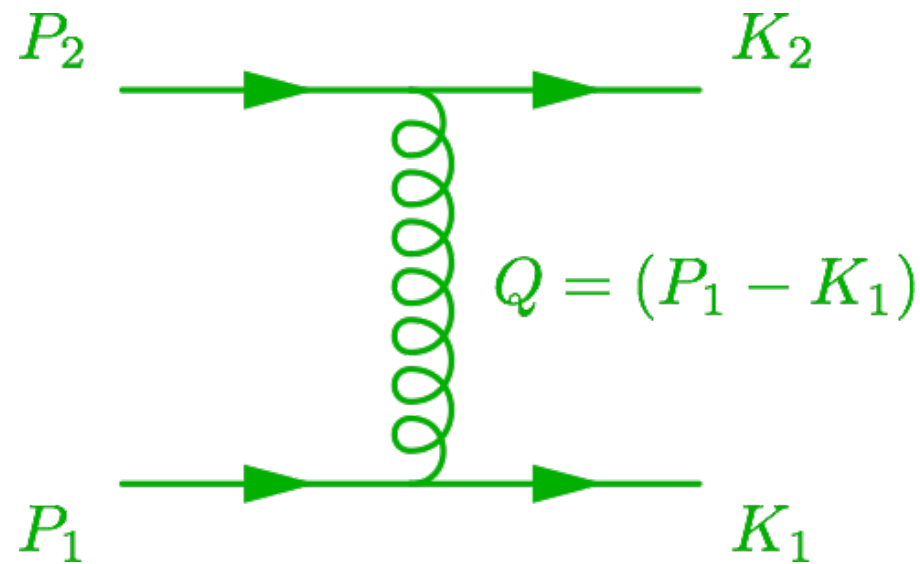




- Quark-Gluon-Plasma (QGP) is formed during Heavy-Ion collisions at RHIC and LHC.
- Different probes of the QGP : collective flow, heavy flavor, EM probes..
- High energetic particles (jets) created at the early stage of the collision can be used as probes to understand the medium.

As the high energetic partons traverse the medium they lose energy due to:

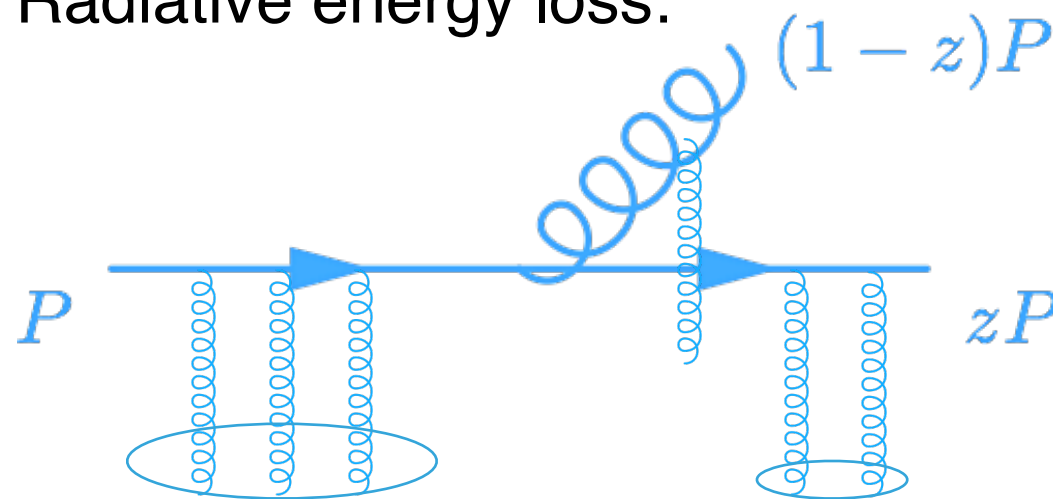
- Elastic energy loss:



Both require input from the medium
transverse scattering rate :

$$\lim_{p \rightarrow \infty} \frac{d\Gamma(\mathbf{p}, \mathbf{p} + \mathbf{q}_\perp)}{d^2 q_\perp} = \frac{\mathcal{C}(q_\perp)}{(2\pi)^2}.$$

- Radiative energy loss:



In the literature one employs pQCD broadening kernels:

- Static screened color centers $\rightarrow C(q) \propto \frac{1}{(q^2 + m_D^2)^2}$
- Dynamics moving charges $\rightarrow C(q) \propto \frac{1}{q^2(q^2 + m_D^2)}$
- Multiple soft scattering $\rightarrow C(b) \propto \frac{\hat{q}}{4} b^2$

Due to the infamous infrared problem of finite temperature QCD

=> perturbative calculation can receive large non-perturbative contribution even at small coupling.

$$C(\mathbf{b}_\perp) \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp}\right) C(q_\perp).$$

The collision kernel can be defined in terms of the behavior of certain null Wilson loops

$$C(\mathbf{b}_\perp) \equiv - \lim_{L \rightarrow \infty} \frac{1}{L} \ln \tilde{W}(L, \mathbf{b}_\perp),$$

[J. Casalderrey-Solana & D. Teaney, JHEP, vol. 04, p. 039, 2007]

=> For temperatures well above T_c these Wilson loops can be recast in the reduced effective theory of electrostatic QCD (EQCD)

[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

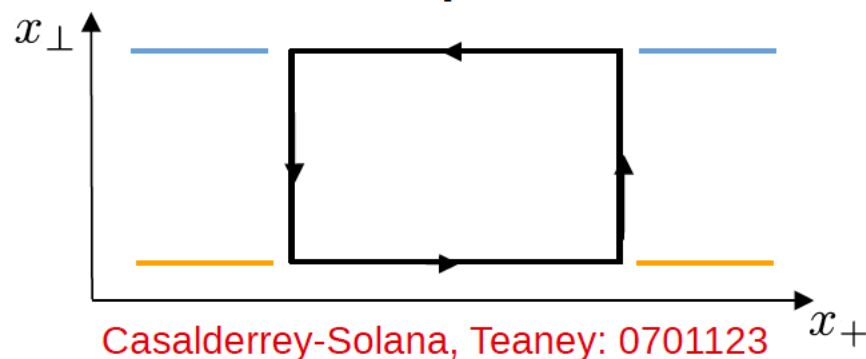
- Collision kernel

$$C(q_{\perp}) = \frac{d\Gamma}{d^2 q_{\perp} dL}$$

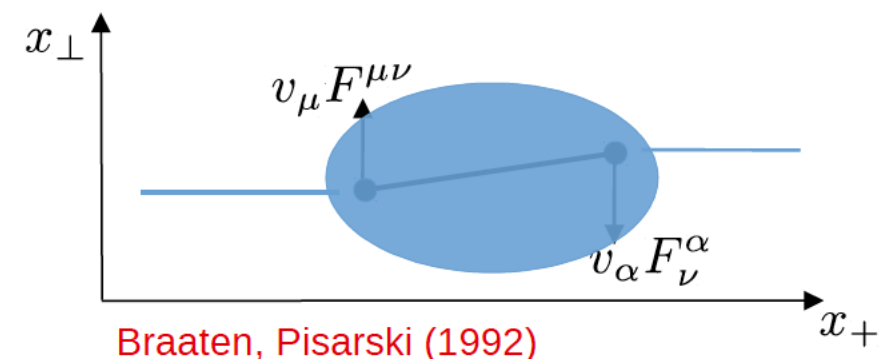
- Asymptotic mass

$$m_{\infty}^2 = C_R (Z_g + Z_f)$$

- Wilson loop



- Force-force-correlator



Nonperturbative gluon-zero-mode contributions:

→ calculate in lattice EQCD

Caron-Huot: 0811.1603

[G.D. Moore & N. Schlusser Phys.Rev.D 102 (2020) 9, 094512]

Courtesy of N. Schlusser

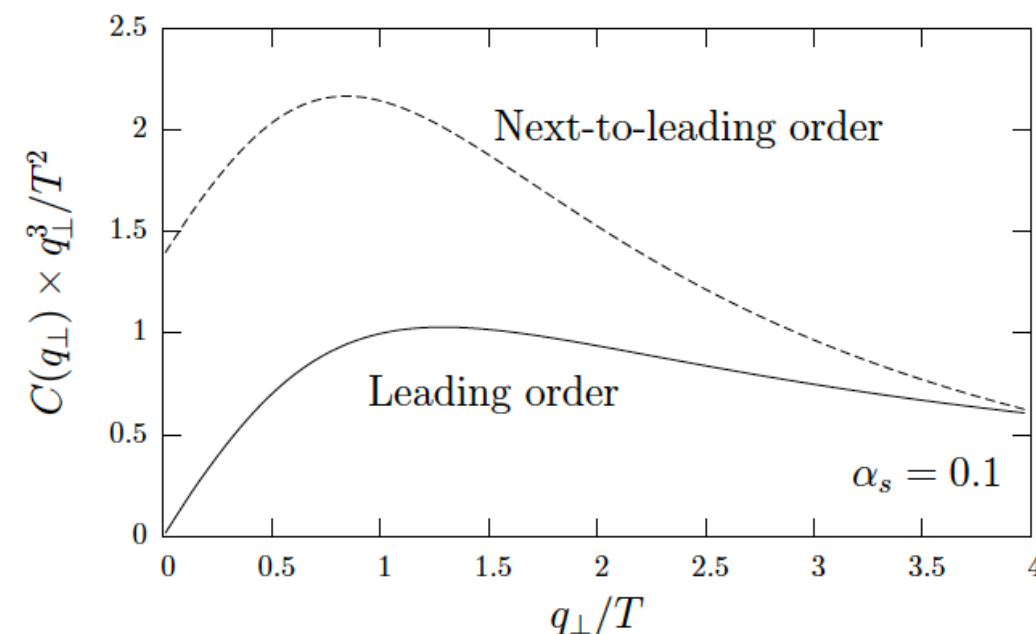
The kernel was computed in a perturbative expansion in the effective theory of EQCD :

- The LO EQCD kernel

$$C_{\text{QCD}}^{\text{LO}}(q_{\perp}) = \frac{g_s^4 T^3 C_R}{q_{\perp}^2 (q_{\perp}^2 + m_D^2)} \int \frac{d^3 p}{(2\pi)^3} \frac{p - p_z}{p} \left[2C_A n_B(p) (1 + n_B(p')) + 4N_f T_f n_F(p) (1 - n_F(p')) \right] = g_s^2 T C_R \begin{cases} \frac{m_D^2 - g_s^2 T^2 C_A \frac{q_{\perp}}{16T}}{q_{\perp}^2 (q_{\perp}^2 + m_D^2)}, & q_{\perp} \ll g_s T, \\ \frac{g_s^2 T}{q_{\perp}^4} \mathcal{N}, & q_{\perp} \gg g_s T, \end{cases}$$

[P. Arnold & W. Xiao Phys.Rev.D 78 (2008), 125008]

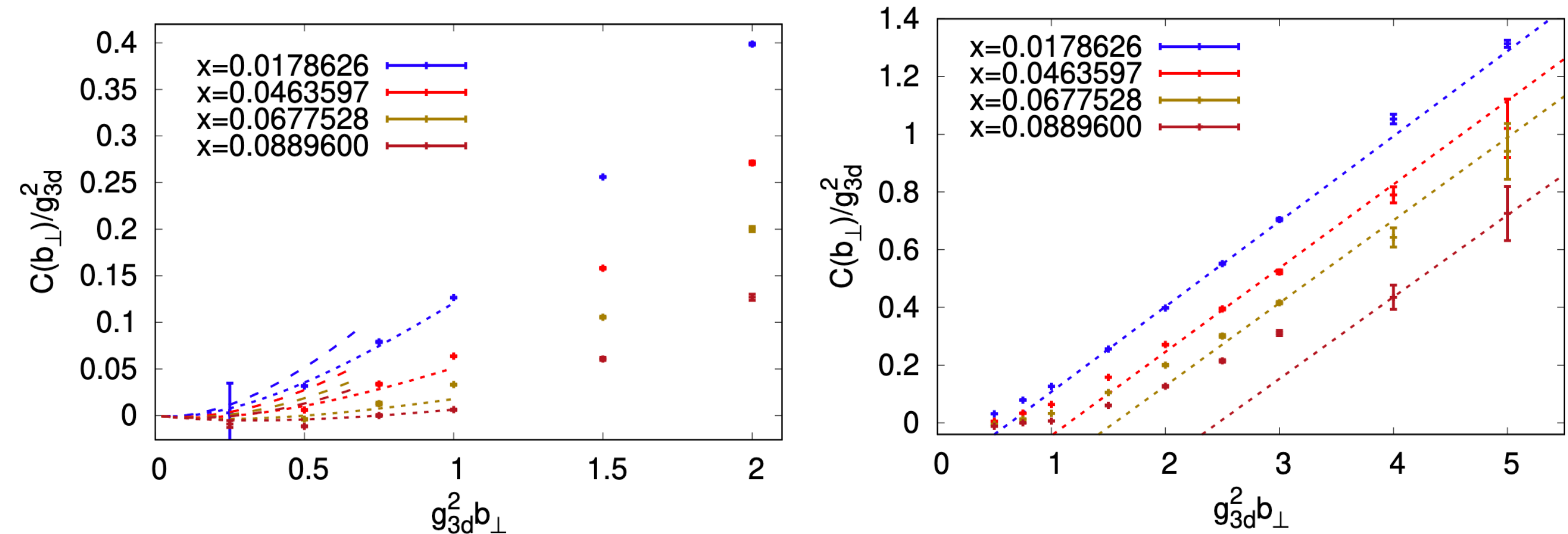
- NLO corrections :



[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]

Beyond the perturbative result, lattice extracted non-perturbative contribution were computed

[G.D. Moore & N. Schlusser Phys.Rev.D 101 (2020) 1, 014505]

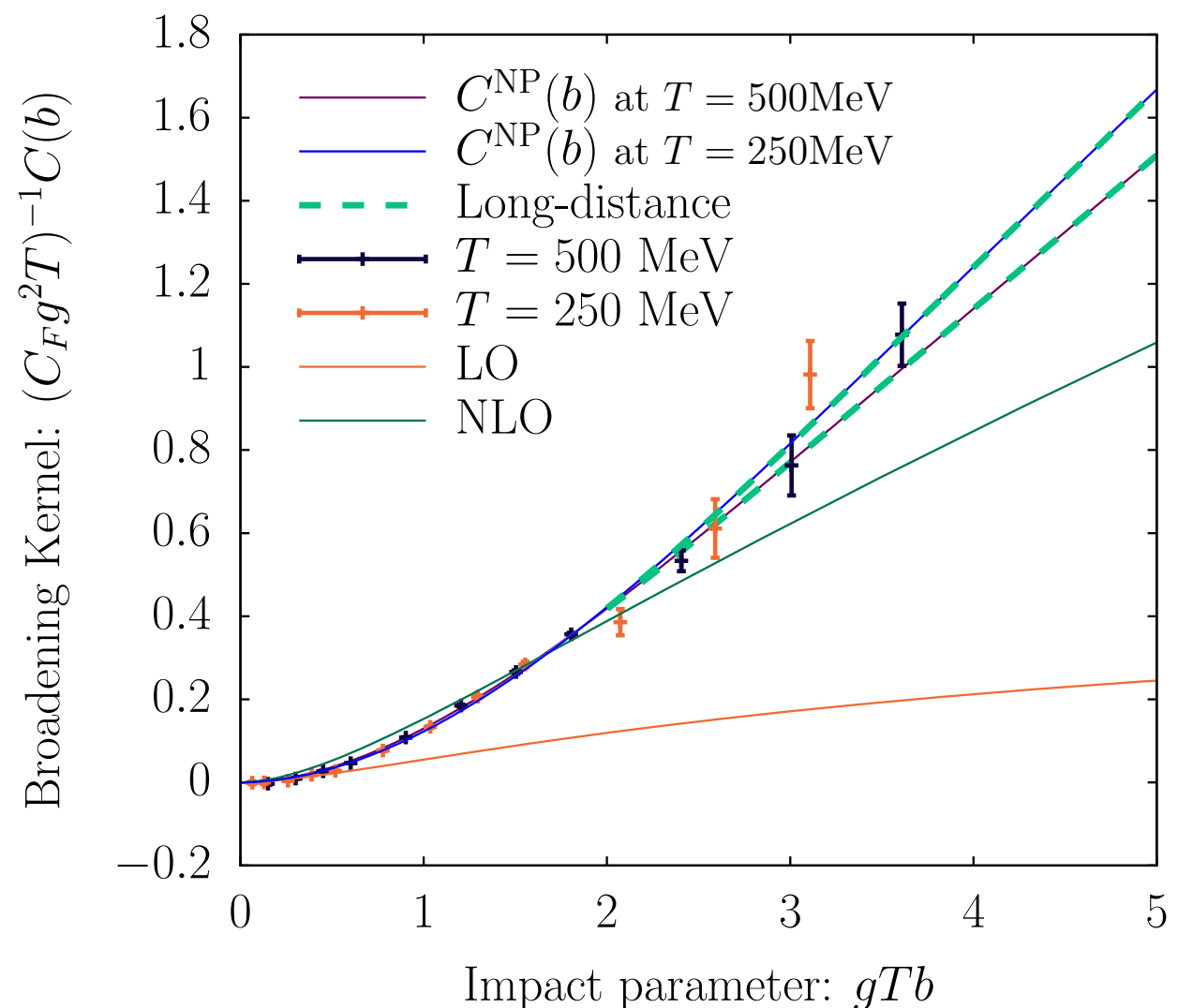
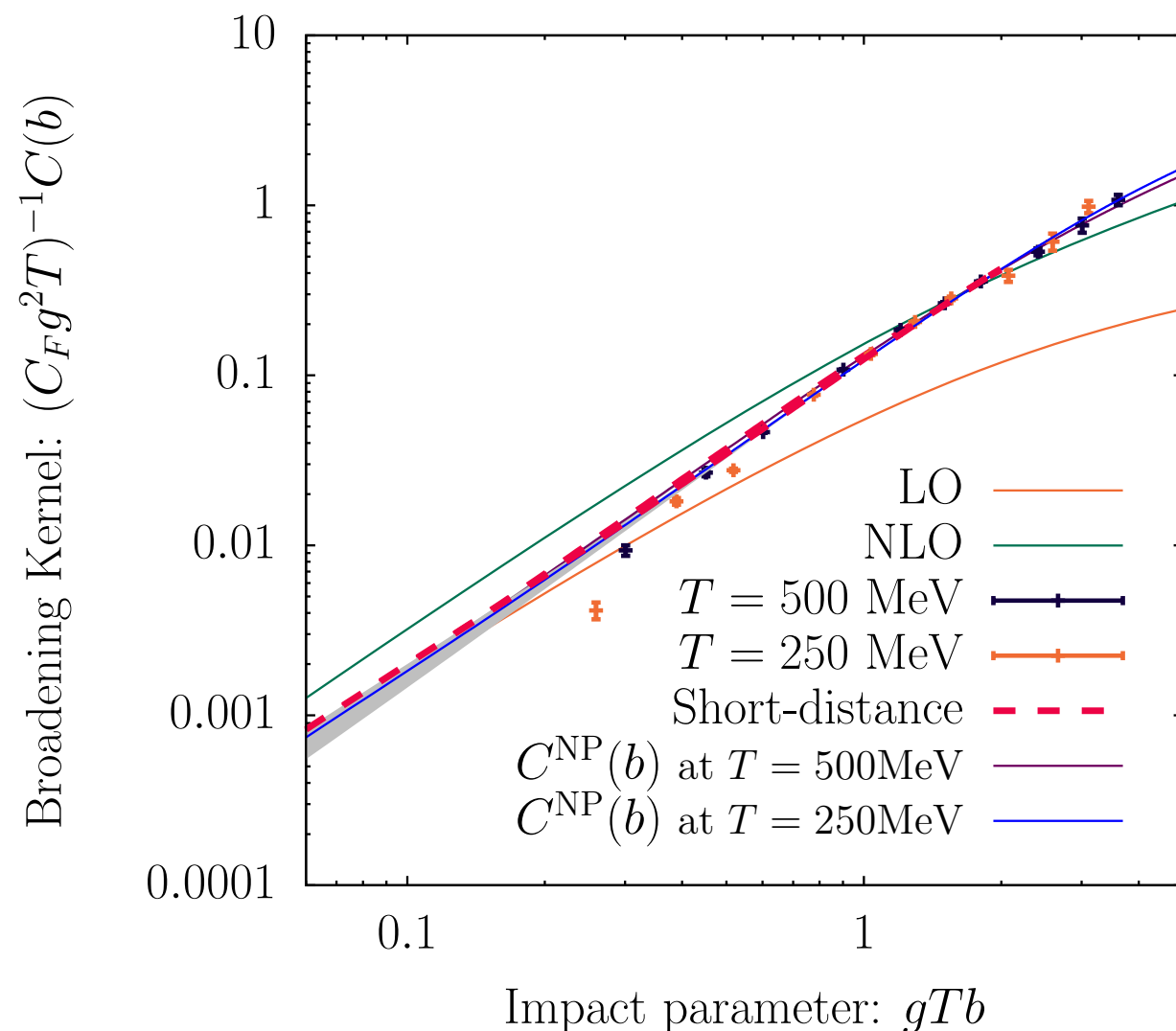


This result here is for the broadening kernel in EQCD which need to be matched to QCD

Since EQCD is a low-energy effective theory for QCD they should both agree in the IR regime but in the UV they can be different.

In order to ensure the right UV behavior while keeping the IR behavior from the lattice result we write the full kernel:

$$C_{\text{QCD}}(b_{\perp}) \approx \left(C_{\text{QCD}}^{\text{pert}}(b_{\perp}) - C_{\text{EQCD}}^{\text{pert}}(b_{\perp}) \right) + C_{\text{EQCD}}^{\text{latt}}(b_{\perp}).$$



[G.D. Moore, S. Schlichting, N. Schlusser, I.S arXiv:2105.01679]

Long-distance behavior :

The kernel follows an area-law with sub-leading logarithm corrections

[M. Laine, Eur. Phys. J. C, vol. 72]

$$\frac{C_{\text{QCD}}}{g_{3\text{d}}^2}(b_{\perp}) \xrightarrow{b_{\perp} \gg 1/g_{3\text{d}}^2} \boxed{A + \frac{\sigma_{\text{EQCD}}}{g_{3\text{d}}^4} g_{3\text{d}}^2 b_{\perp}} + \frac{g_{\text{s}}^4 C_{\text{R}}}{\pi} \left[\frac{y}{4} \left(\frac{1}{6} - \frac{1}{\pi^2} \right) + \frac{C_{\text{A}}}{8\pi^2 g_{\text{s}}^2} \right] \log(g_{3\text{d}}^2 b_{\perp}),$$

Short-distance behavior :

The kernel follows the same behavior as the LO one, where we determine $\hat{q}_0$ from the data :

$$\frac{C_{\text{QCD}}}{g_{3\text{d}}^2}(b_{\perp}) \xrightarrow{b_{\perp} \ll 1/m_{\text{D}}} -\frac{C_{\text{R}}}{8\pi} \frac{\zeta(3)}{\zeta(2)} \left(-\frac{1}{2g_{\text{s}}^2} + \frac{3y}{2} \right) (g_{3\text{d}}^2 b_{\perp})^2 \log(g_{3\text{d}}^2 b_{\perp}) + \frac{1}{4} \frac{\hat{q}_0}{g_{3\text{d}}^6} (g_{3\text{d}}^2 b_{\perp})^2,$$

Using the broadening kernel at hand, one can compute in-medium splitting rates

- In the AMY approach:

$$\frac{d\Gamma_{ij}}{dz}(P, z) = \frac{\alpha_s P_{ij}(z)}{[2Pz(1-z)]^2} \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \text{Re} [2\mathbf{p}_\perp \cdot \mathbf{g}_{(z,P)}(\mathbf{p}_\perp)]$$

where $g_{(z,P)}(p_\perp)$ is solution to the integral equation :

$$\begin{aligned} 2\mathbf{p}_\perp = i\delta E(z, P, \mathbf{p}_\perp) \mathbf{g}_{(z,P)}(\mathbf{p}_\perp) &+ \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} \bar{C}(q_\perp) \\ &\times \left\{ C_1 [\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - \mathbf{q}_\perp)] \right. \\ &\quad + C_z [\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - z\mathbf{q}_\perp)] \\ &\quad \left. + C_{1-z} [\mathbf{g}_{(z,P)}(\mathbf{p}_\perp) - \mathbf{g}_{(z,P)}(\mathbf{p}_\perp - (1-z)\mathbf{q}_\perp)] \right\} . \end{aligned}$$

BH regime :

$$Pz(1-z) \ll \omega_{\text{BH}} \sim T$$

Formation time is small and interference between scatterings can be neglected.

One solves the rate equations in opacity expansion in the number of elastic scatterings with medium.

Amounts to calculating a numerical integral.

LPM regime :

$$Pz(1-z) \gg \omega_{\text{BH}} \sim T$$

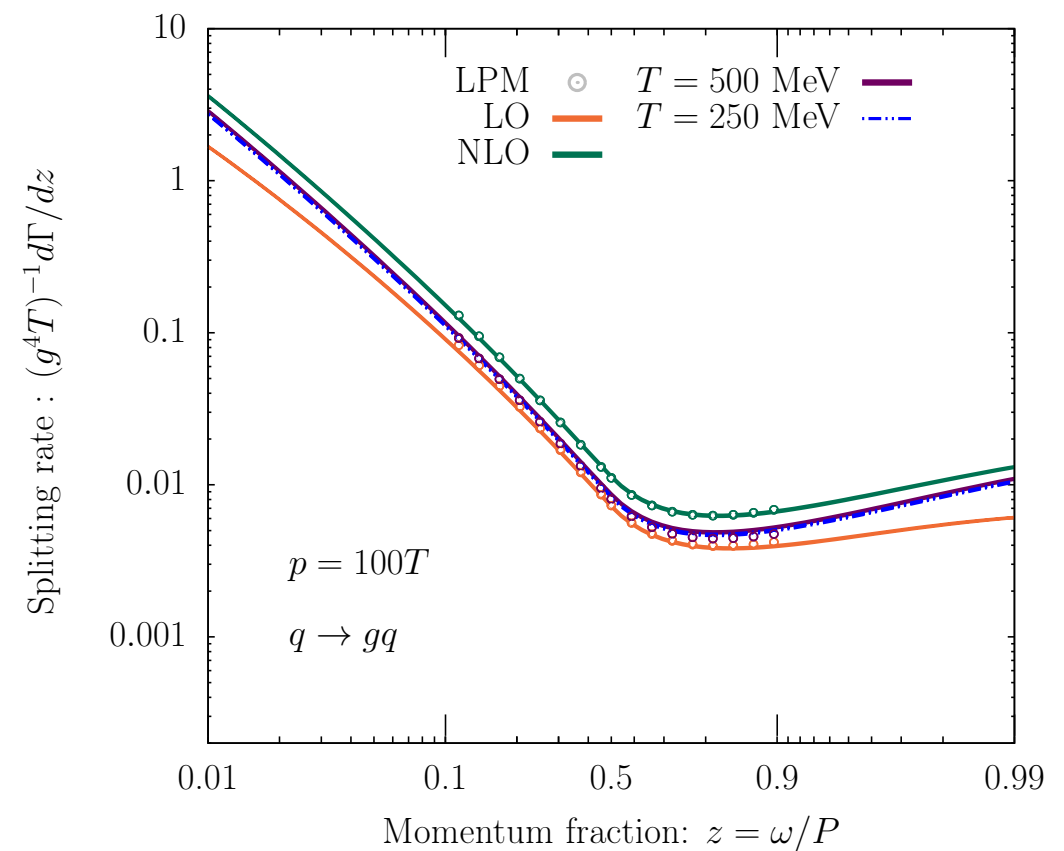
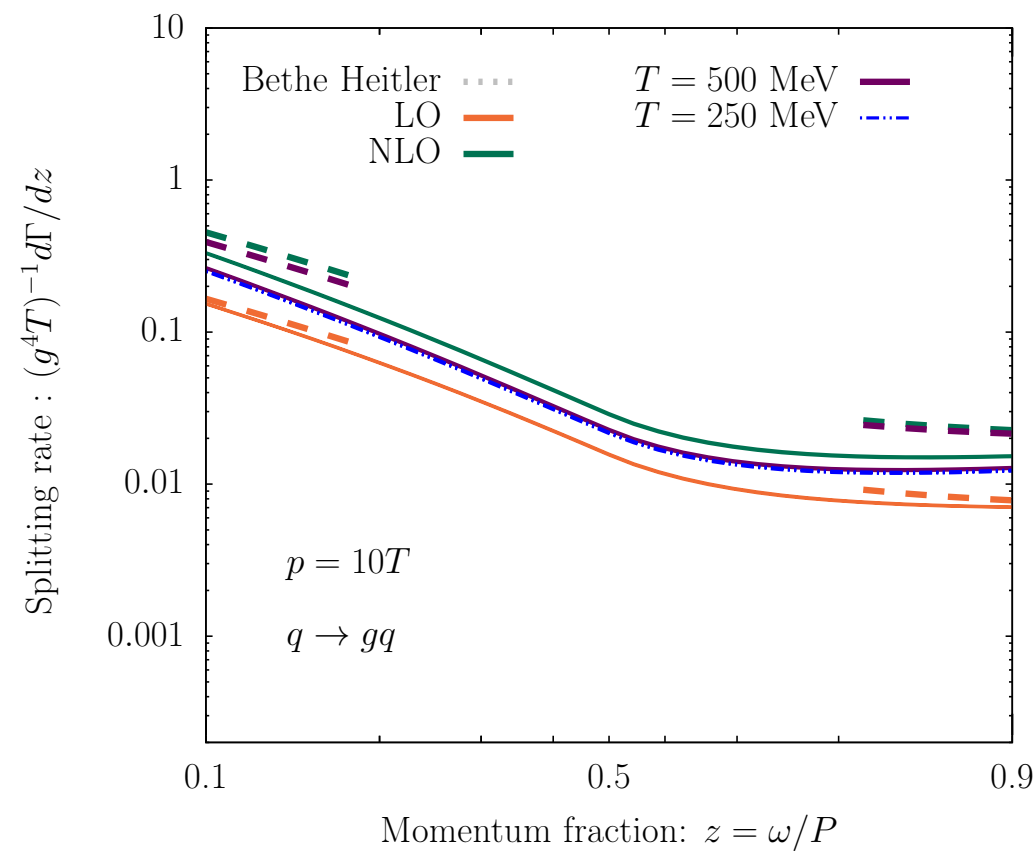
Typical number of rescatterings within the formation time of bremsstrahlung can be large.

Interference of many soft scatterings need to be considered.

The broadening kernel follows a $b_{\perp}^2$ behavior at small distances

$$C(b_{\perp}) = -\frac{g_s^2 T^2}{16\pi} \mathcal{N} b_{\perp}^2 \log(\xi m_D^2 b_{\perp}^2 / 4)$$

The rate equations become a Harmonic oscillator problem and solved analytically



- The two temperatures (250 MeV and 500 MeV) do not display a remarkable difference
- Recover LPM suppression at large momentum.
- In the large energy region the rate is closer to the LO one as they both follow the same behavior in broadening kernel at short distances

Finite medium

To compute the rate in finite medium it is best to work in momentum space

The rate follows the limits :

- $q \gg 1$:

$$C^{\text{UV}}(q_{\perp}) = \frac{C_R}{8\pi} \frac{\zeta(3)}{\zeta(2)} \left(-\frac{1}{2g_s^2} + \frac{3y}{2} \right) \frac{8\pi}{q_{\perp}^4}.$$

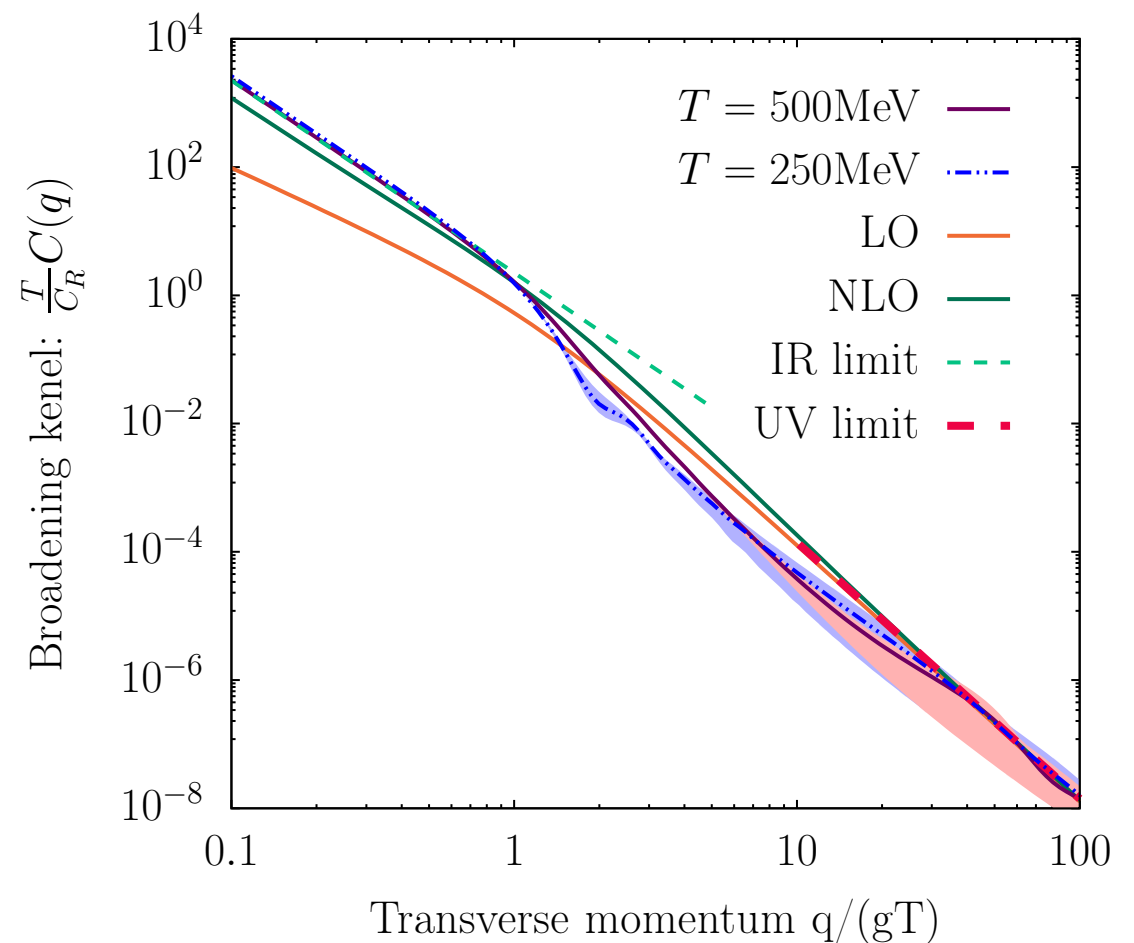
Similar to the LO rate

- $q \ll 1$:

$$C^{\text{IR}}(q_{\perp}) = \frac{2\pi}{q_{\perp}^3} \frac{\sigma_{\text{EQCD}}}{g_{3d}^2} + \frac{g^4 C_R}{\pi} \left[\frac{y}{4} \left(\frac{1}{6} - \frac{1}{\pi^2} \right) + \frac{C_A}{8\pi^2 g_s^2} \right] \frac{2\pi}{q_{\perp}^2}. \quad (6)$$

stems from the area law with string tension

Transverse Momentum Broadening:



- We compare the finite medium with an improved opacity expansion where after we cut low momentum interactions, we exponentiate higher order of the expansion

$$\frac{d\Gamma_{bc}^a}{dz} = \frac{g^2 P_{bc}^a(z)}{\pi P} \text{Re} \int_p \frac{1 - e^{-(i\delta E(\mathbf{p}) + \Sigma(\mathbf{p}^2))t}}{i\delta E(\mathbf{p}) + \Sigma(\mathbf{p}^2)} \psi^{(1)}(\mathbf{p}) .$$

where the first order wave function is the collision integral of the initial condition

$$\begin{aligned} \psi^{(1)}(\mathbf{p}) = \int_q n(t) C(\mathbf{q}) \left\{ C_1 \left[\frac{p^2}{\epsilon(\mathbf{p})} - \frac{p^2 - \mathbf{p} \cdot \mathbf{q}}{\epsilon(\mathbf{p} - \mathbf{q})} \right] \right. \\ \left. + C_z \left[\frac{p^2}{\epsilon(\mathbf{p})} - \frac{p^2 + z\mathbf{p} \cdot \mathbf{q}}{\epsilon(\mathbf{p} + z\mathbf{q})} \right] + C_{1-z} \left[\frac{p^2}{\epsilon(\mathbf{p})} - \frac{p^2 + (1-z)\mathbf{p} \cdot \mathbf{q}}{\epsilon(\mathbf{p} + (1-z)\mathbf{q})} \right] \right\} \end{aligned}$$

and the subsequent interactions are encoded in the exponential of

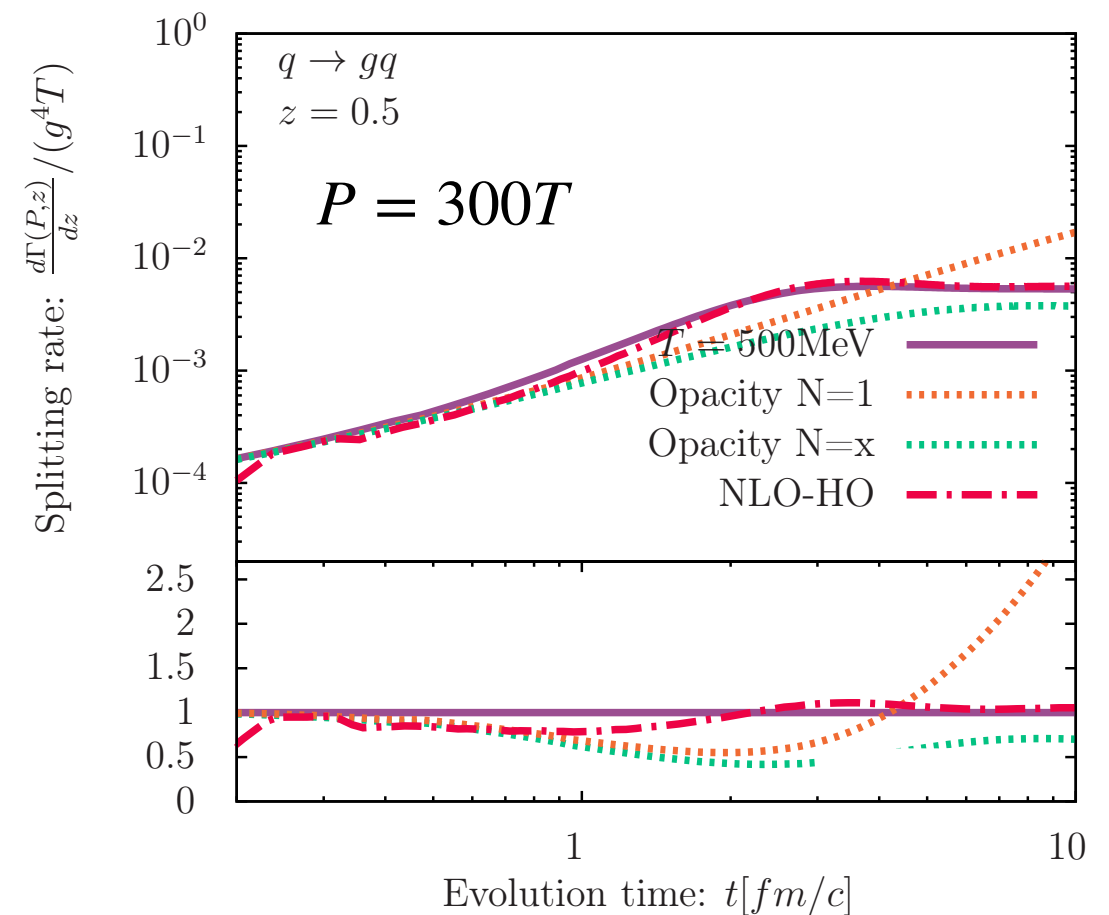
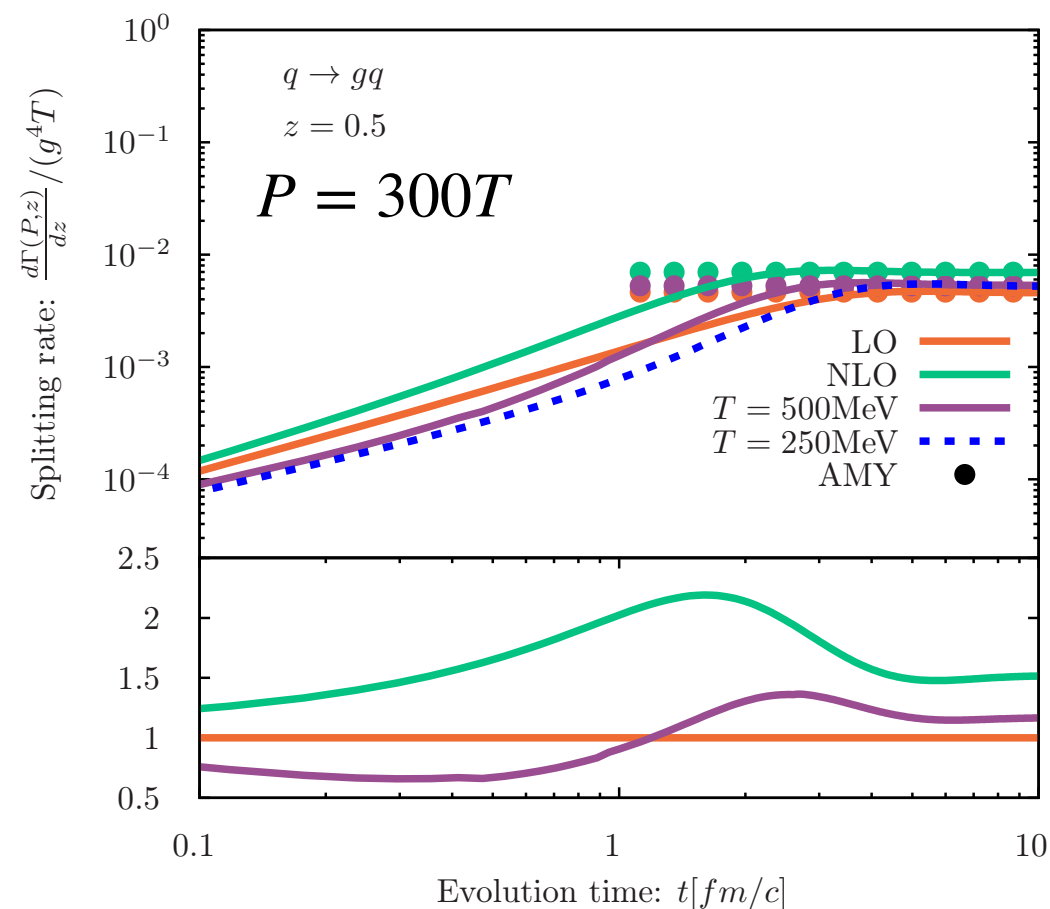
$$\Sigma(M^2) \equiv \int_{\vec{k}^2 > M^2} n(t) C(\vec{k}) (C_1 + C_z + C_{1-z})$$

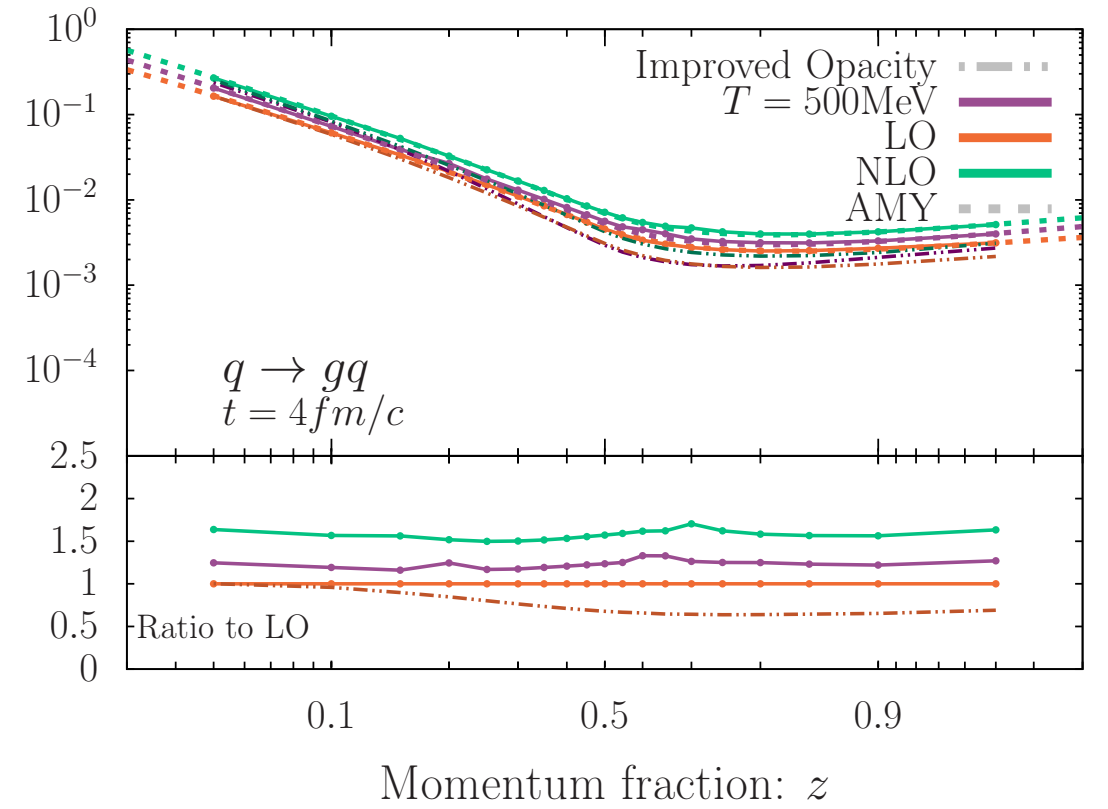
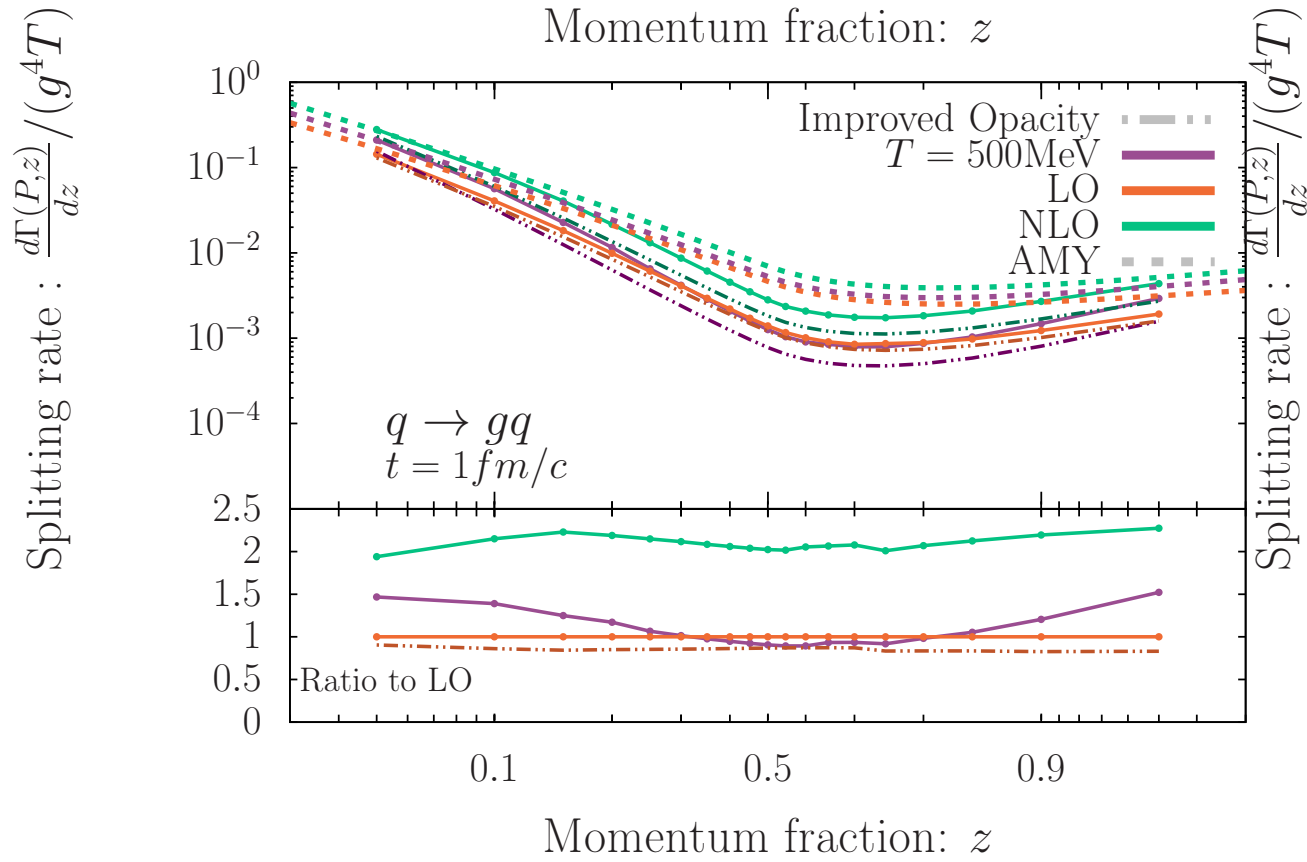
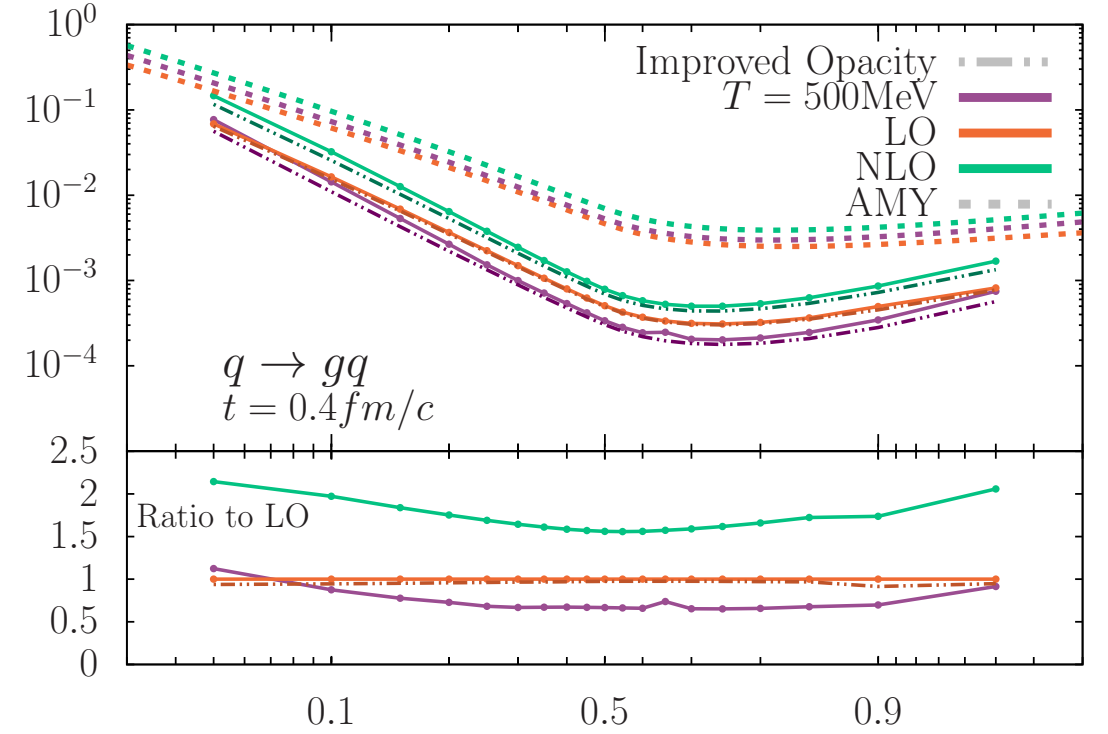
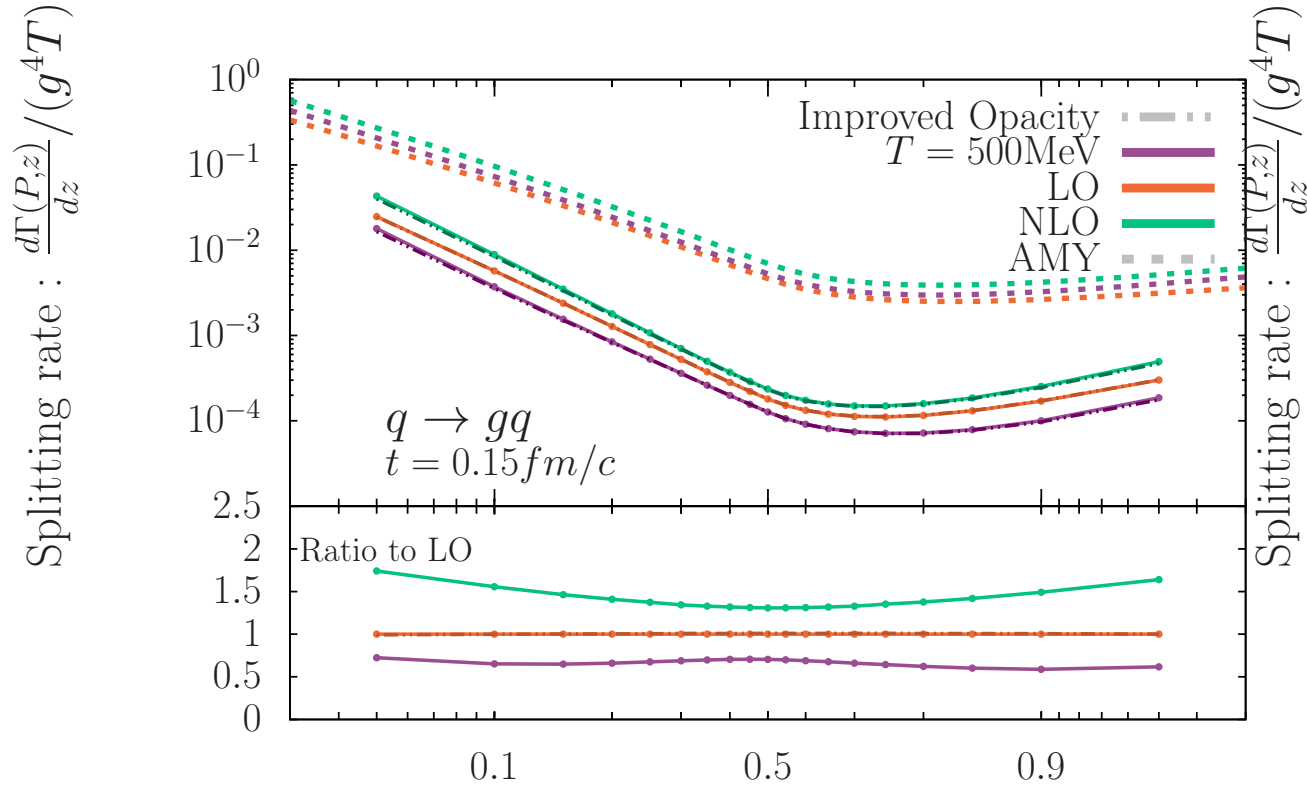
[C. Andres et Al. JHEP 03 (2021), 102]

- We compute the rate in finite-medium following the approach of S. Caron-Huot and C. Gale

[S. Caron-Huot, C. Gale Phys.Rev.C 82 (2010), 064902]

Medium-induced splitting rates:





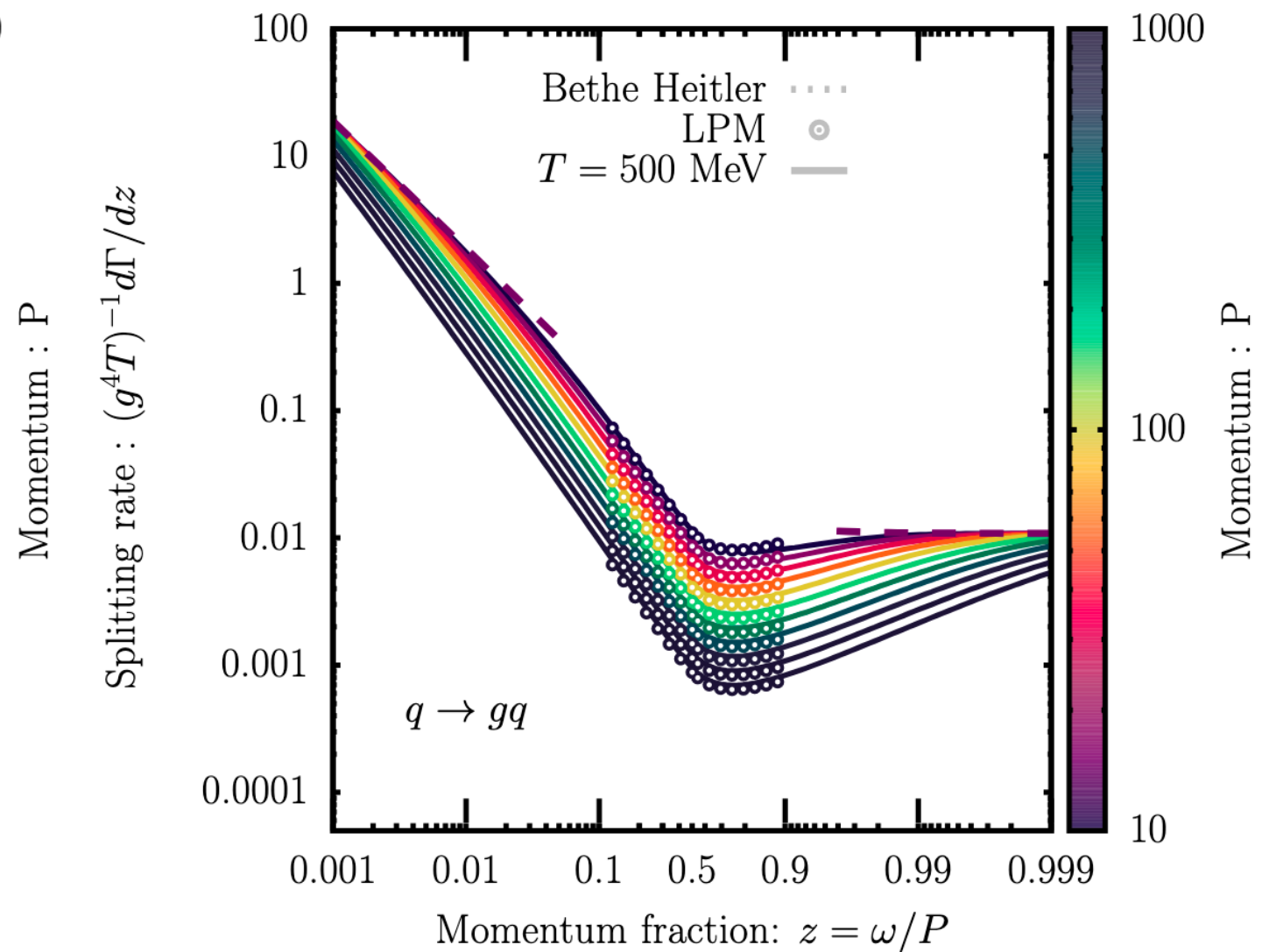
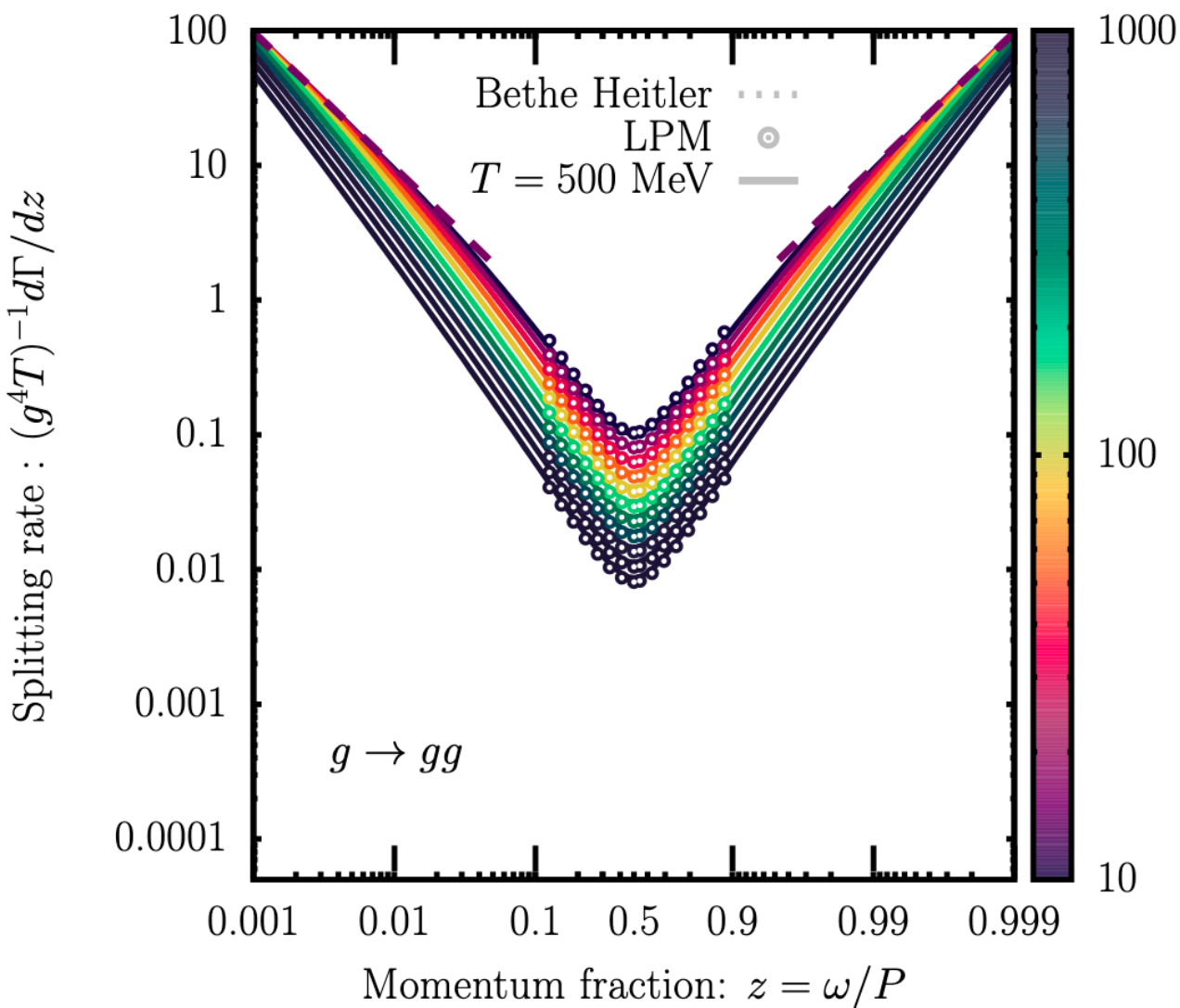
- Using the non-perturbative contribution, we observe a sizable difference to the perturbative broadening kernel as well as to the in-medium splitting rates ($\sim 25\%$)
- Although the limiting behaviors are recovered in the limiting cases, a robust full calculation of the rate is important.

[G.D. Moore & N. Schlusser Phys.Rev.D 102 (2020) 9, 094512]

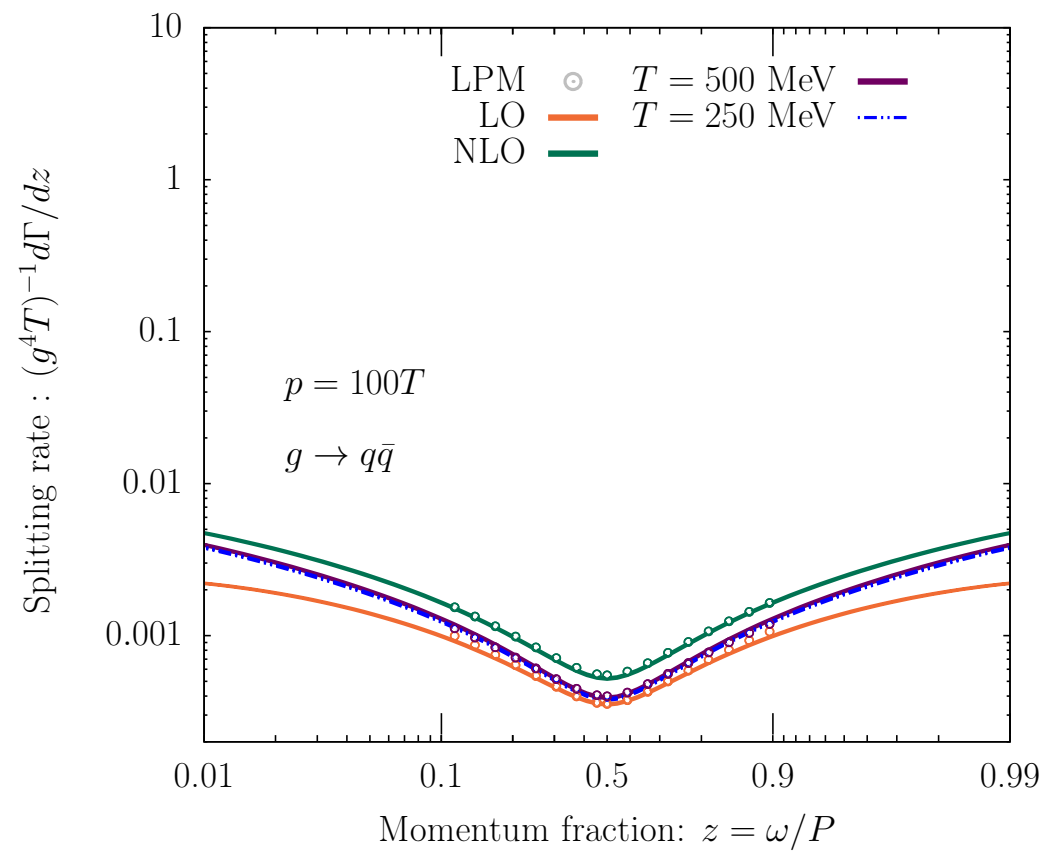
- There have been effort to extract asymptotic masses using the same procedure which still needs to be matched to QCD.
- Would be interesting to include the non-perturbative results to jet studies (elastic and radiative interactions) in kinetic studies or MC

Thank you!

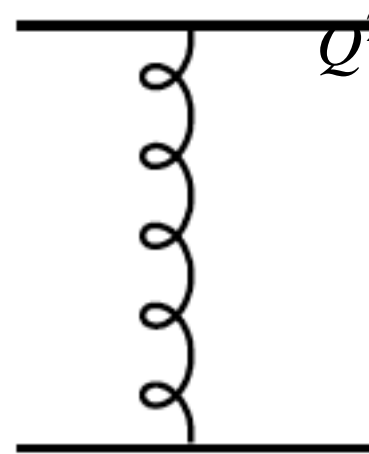
Backup



- BH regime at small energies



- Elastic interaction with the thermal bath are well described by perturbative theory at large $q_{\perp} \gg gT$
- But when we consider smaller $q_{\perp} \sim gT$ we require non-perturbative input.



$Q^2 = \omega^2 - q^2$

- For temperatures well above T_c , one can use Electrostatic QCD to compute the elastic broadening kernel.

[P. Arnold & W. Xiao Phys.Rev.D 78 (2008), 125008]

[S. Caron-Huot Phys.Rev.D 79 (2009), 065039]